NATIONAL TRANSPORTATION SAFETY BOARD
Vehicle Recorder Division
Washington, D.C. 20594

July 16, 2008

Gusset Plate Photographs – Measurement Study Report

NTSB Accident Number:
HWY07MH024

by Doug Brazy

A. ACCIDENT

Location: Minneapolis, MN
Date: August 1, 2007
Time: 1805 Central Daylight Time
Event: Collapse of I-35W Bridge

B. GROUP

N/A

C. SUMMARY

On Wednesday, August 1, 2007, about 6:05 p.m. central daylight time, the Interstate 35W (I-35W) highway bridge over the Mississippi River in Minneapolis, Minnesota, collapsed.

Several precollapse photographs of the bridge truss that show the U10 and U10 prime nodes (both East and West)[1], were provided to the Vehicle Recorder Division for analysis. These photographs depict a "bowing" or out of plane distortion along the ends of several gusset plates, in the span between the upper chord members and the L9/U10 diagonal members that

[1] See the Structural Investigation Group Chairman Factual Report for a description of the bridge and numbering system for the nodes of the deck truss portion of the bridge. All photographs discussed in this report can be found in the public docket under a separate cover entitled "Pre-Collapse Images – Nodes U10 and L11".

enter the nodes. Figure 1 is a photograph taken in August 1999 which shows the inner, or western, gusset plate at node U10 East.

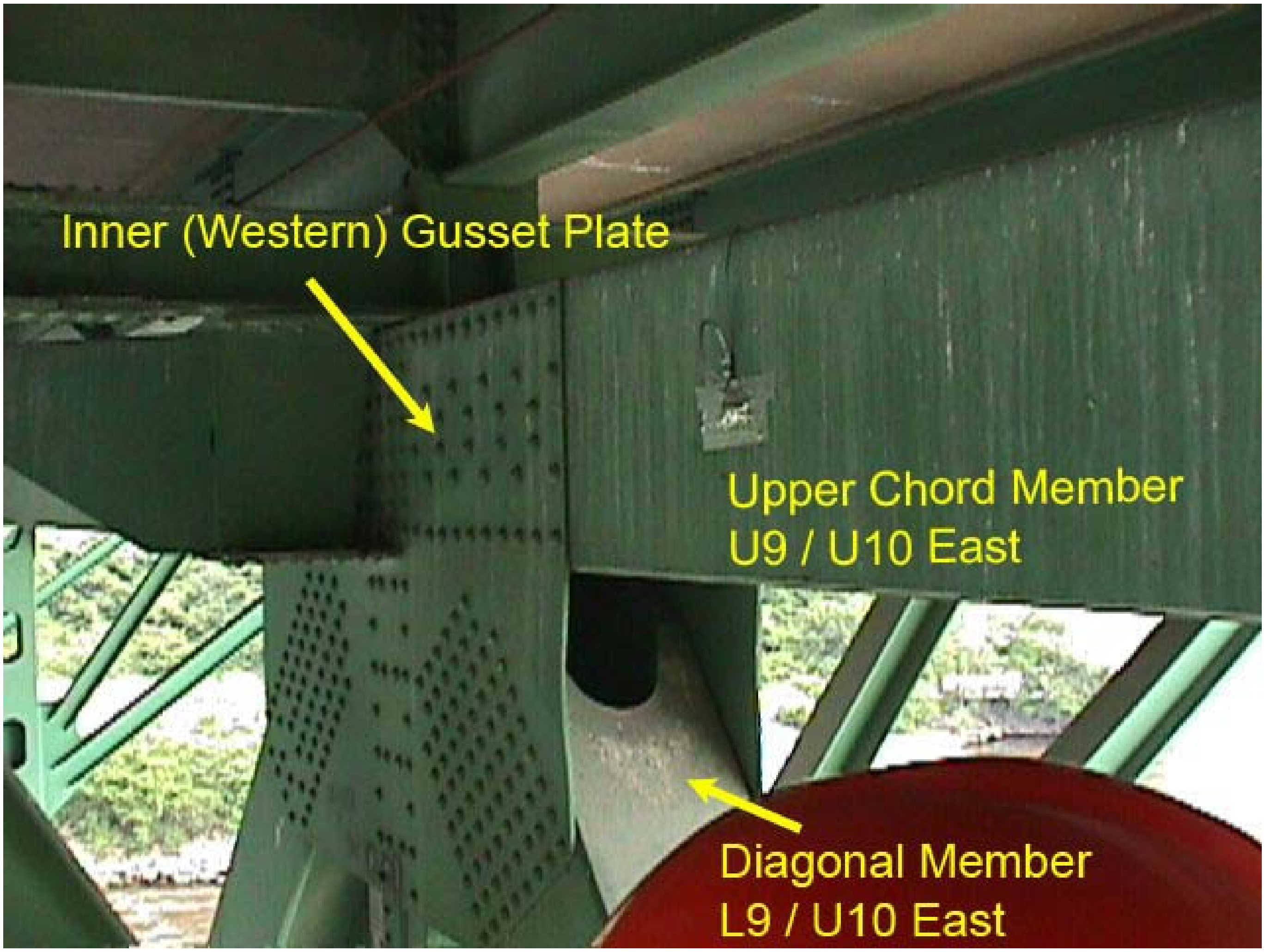

Figure 1 - Node U10 East - Photo MVC-010F.JPG

In Figure 1, the vertical ½ inch wide end of the western gusset plate appears to be bowed towards the left. Figure 2 shows the same photograph with a straight reference line (white) overlaid along the eastern (right) edge of the plate end. The white line shows where this eastern edge would be if the plate were not bowed.

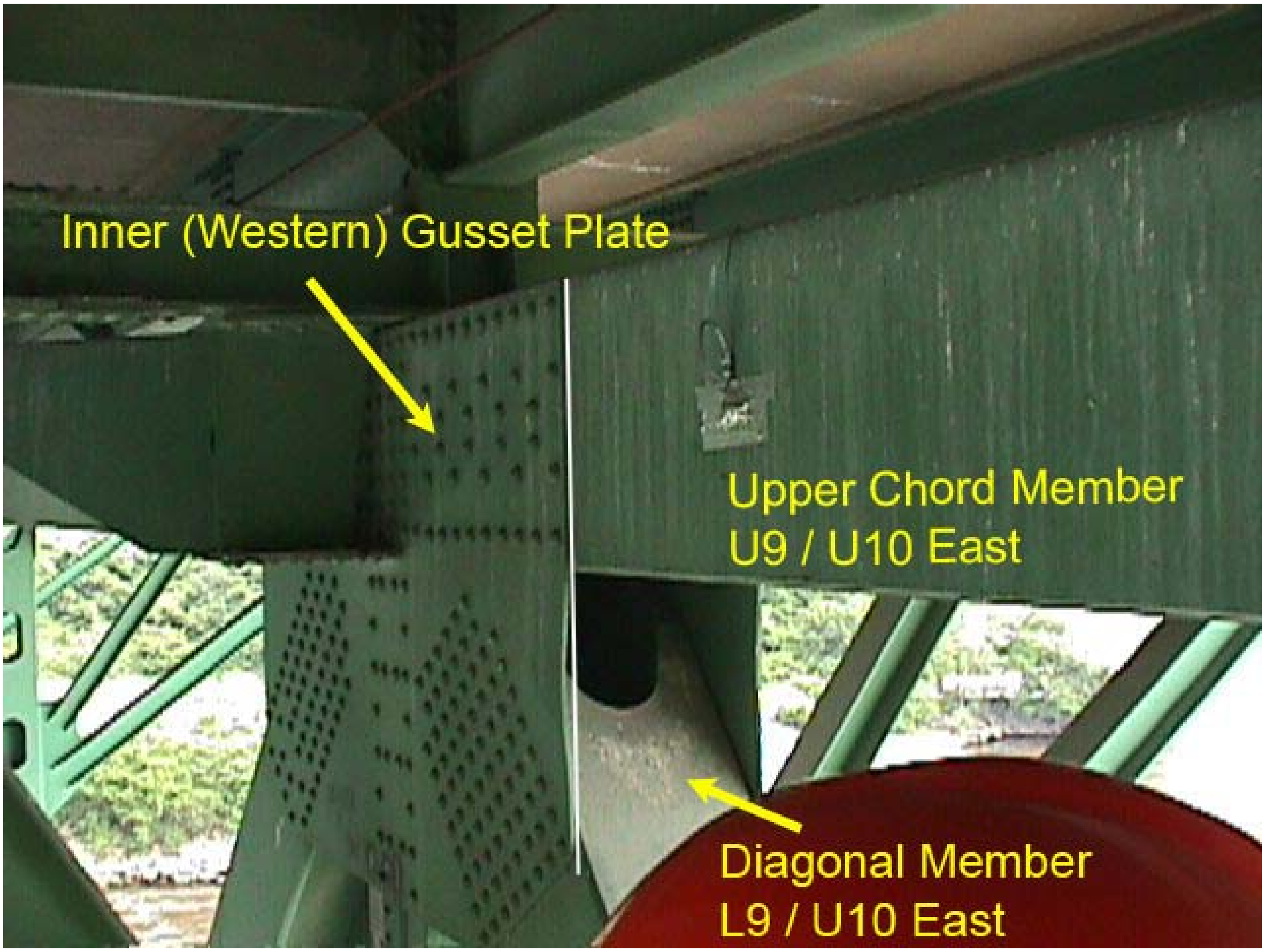

Figure 2 - Edge Reference Line

This photograph and several others were examined in an effort to calculate the maximum displacement in the gusset plate ends. The results indicate that the western (inner) gusset plate at node U10 East, was displaced between 0.69 and 0.81 inch (towards the west), at its point of maximum deflection. Similarly, the eastern (inner) gusset plate at node U10 West, was displaced between 0.46 and 0.64 inch at its point of maximum deflection (also towards the west). Two different photographs of U10 East, taken nearly 4 years apart, indicate that the amount of bowing did not change over that 4 year period (August 1999, to June 2003).

The western (inner) gusset plate at node U10′ East was displaced between 0.49 to 0.60 inch. The eastern (inner) gusset plate at node U10′ West was displaced between 0.82 an 0.99 inch. Both of these displacements were in the opposite direction of the bowing at the U10 nodes (i.e. the bowing at the U10′ nodes was toward the east). Table 1 at the end of this report contains a summary of all calculations.

The displacement was only calculated for the inner gusset plates at the U10 and U10′ nodes. However some amount of displacement could be seen in the outer gusset plate ends as well. Out of the total of 16 gusset plate ends at these nodes, some displacement could be detected in 7 of them. All observed displaced edges were between the compression diagonal (L9/U10 or L9′/U10′) and the upper chord, and none of the edges between the tension diagonal (U10/L11 or U10′/L11′) and the upper chord had observable displacement. Five edges appeared to have no displacement, two edges were too unclear to determine if any displacement had occurred, and two edges did not appear in any of the photographs.

D. DETAILS OF INVESTIGATION

A total of 7 photographs were examined in detail. Two photos of the U10 East node as shown in Figures 3 and 4[2];

Figure 3 (Image MVC-010F.JPG)

Figure 4 (image DSK9-29S.jpg)

and 3 photos of the U10 West node, as shown in Figures 5-7;

[2] The photograph in Figure 3 was taken by University of Minnesota staff in August 1999, as part of a project in which strain gauges were applied to the U10-U9 and U10-L9 truss members on both the east and west trusses (the gauges were protected by tape and are visible in Figures 3-7). The photographs in Figures 4 through 9 were taken by URS, an engineering firm, in June 2003 as part of a fatigue evaluation that was conducted under contract to the Minnesota Department of Transportation.

Figure 5 (image DSK5-02S.JPG)

Figure 6 (image DSK5-03S.JPG)

Figure 7(image DSK5-04S.JPG)

and one image each for nodes U10′ East and U10′ West as seen in Figures 8 and 9.

Figure 8 (image DSK9-02S.JPG)

Figure 9 (image DSK4-04S.jpg)

The methodology used to extract measurements from the photos was, in general:

- Construct reference lines on the image in order to establish perspective
- Construct measurement reference lines within the established perspective (marking the features seen in the photo to be measured)
- Establish an appropriate scale using feature(s) of known dimensions
- Measure the out of plane displacement of the gusset plate end, at its maximum deviation from a flat reference plane.

Two separate approaches were taken based on these elements, and each approach yielded similar but slightly different results for each image. The first approach uses a ‘direct scaling’ method, the second uses a ‘dimensional ratio’ method. Image MVC-010F.JPG (Figure 3) showing node U10 East, is used as an example in this report to demonstrate the two approaches used to calculate the amount of gusset plate displacement. The same methodology was applied to the other photographs.

Direct Scaling Method

The direct scaling method utilizes measurements taken directly from the photograph, an appropriate scale using known dimensions, and any necessary corrections to account for the camera’s perspective view.

Measurements are made by constructing reference lines on the photograph which can be used to establish the perspective, and mark the items to be measured. Each photo was imported into a computer aided drafting software program, in order to construct reference lines and take measurements directly from the image.

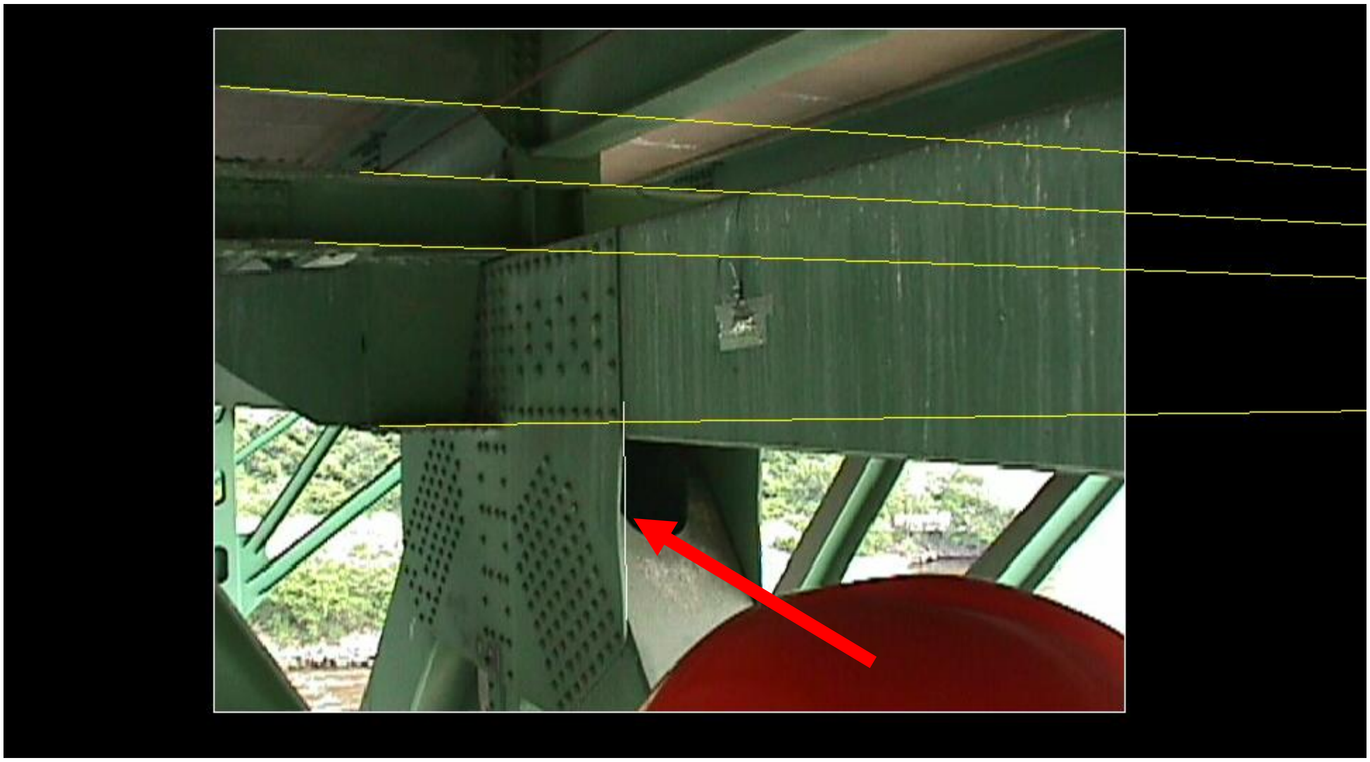

Figure 10 – Reference Lines

Establish Perspective and Orientation

Vertical Reference

In Figure 10, the white line denoted by the red arrow shows where the gusset plate's edge should be, if the plate was not bowed. This line represents the "right" side of the ½" thick end of gusset plate (when viewed from the south). This line was constructed by aligning it with the straight portions of the plate edge where it is flush with the upper chord member at the top, and flush with the diagonal member at the bottom. This line will also serve as a measurement line, from which to reference the displacement of the gusset plate. [3]

[3] Reference and measurement lines may appear "jagged" as seen in the figures in this report. This is an artifact of rastering of the image when exported from the CAD software program. These artifacts to not affect the accuracy of the measurements taken.

Lateral Reference

The yellow lines in Figure 10 were drawn along features that are truly parallel to one another, and oriented normal to the plane of the gusset plate. These lines converge due to the camera's perspective view of these features. Each line was then extended past the edge of the photograph to the convergence location, referred to as a "vanishing point" (VP). See Figure 11.

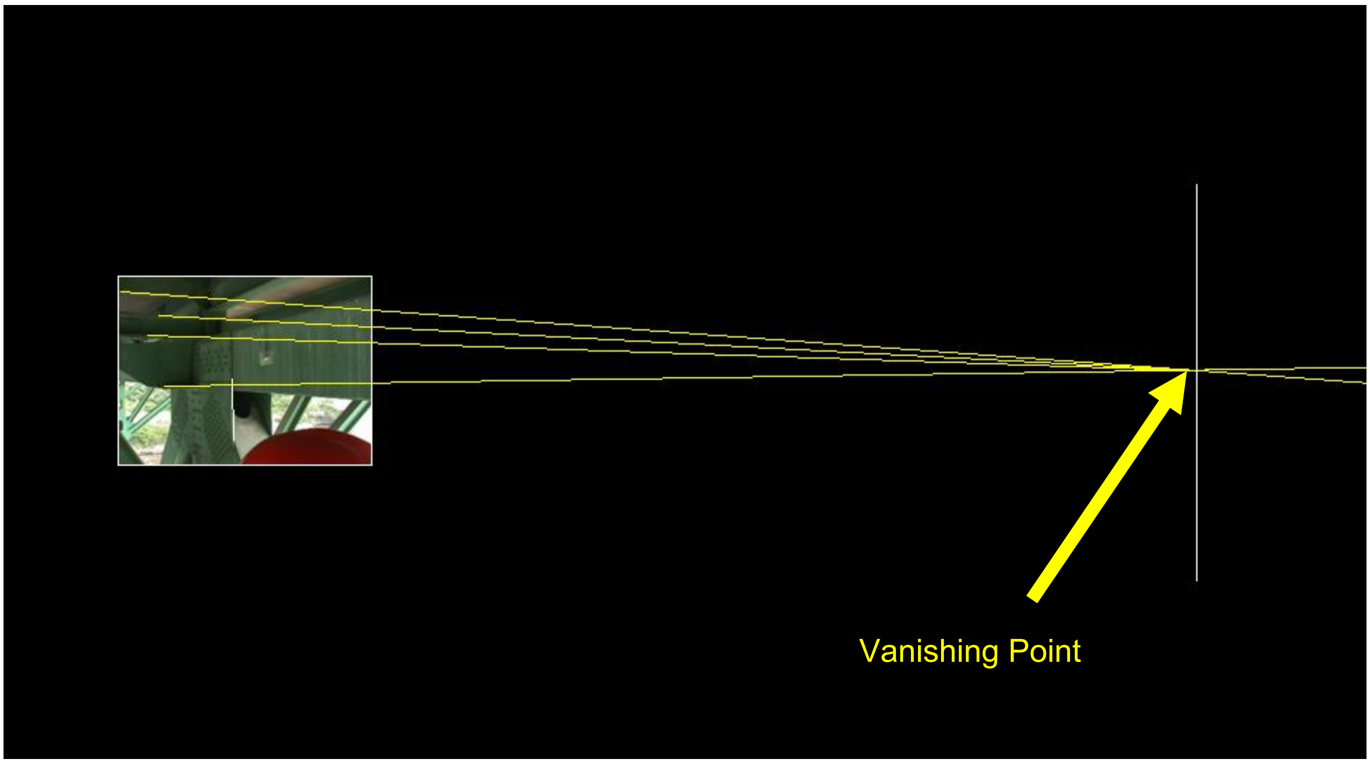

Figure 11 - Vanishing Point

Once the vanishing point is established, reference lines can be constructed that originate at the VP, and placed anywhere in the image to establish a convenient lateral reference point that is parallel to the original known (visible) lateral references.

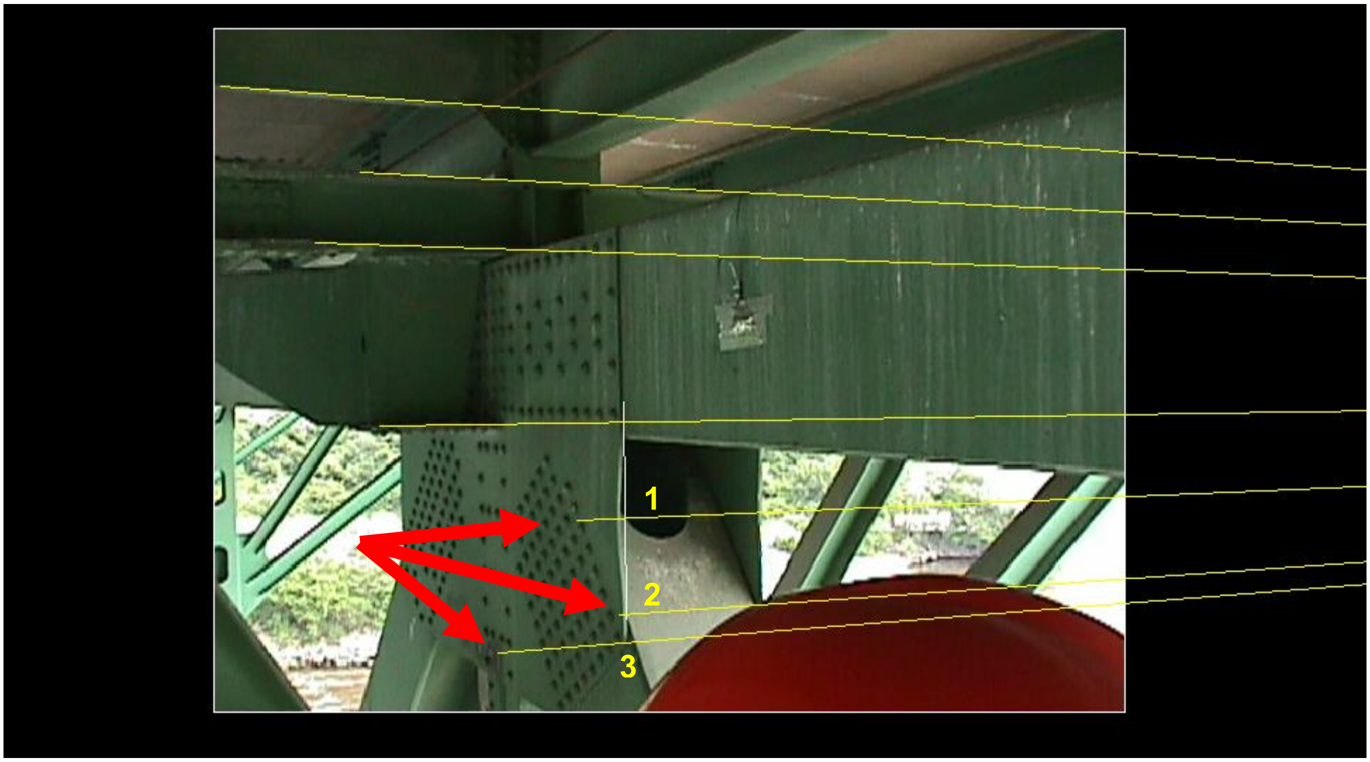

Figure 12 - New Lateral References

In Figure 12, three additional lateral references were added by constructing lines from the VP to (1) the location of the maximum deflection of the gusset plate end, (2) the intersection of the gusset plate edge with the top face of diagonal member L9 / U10 East, and (3) the corner of the gusset plate.

<u>Measurement Lines</u>

Figure 13 is an enlarged view of the gusset plate end, at the location of maximum displacement.

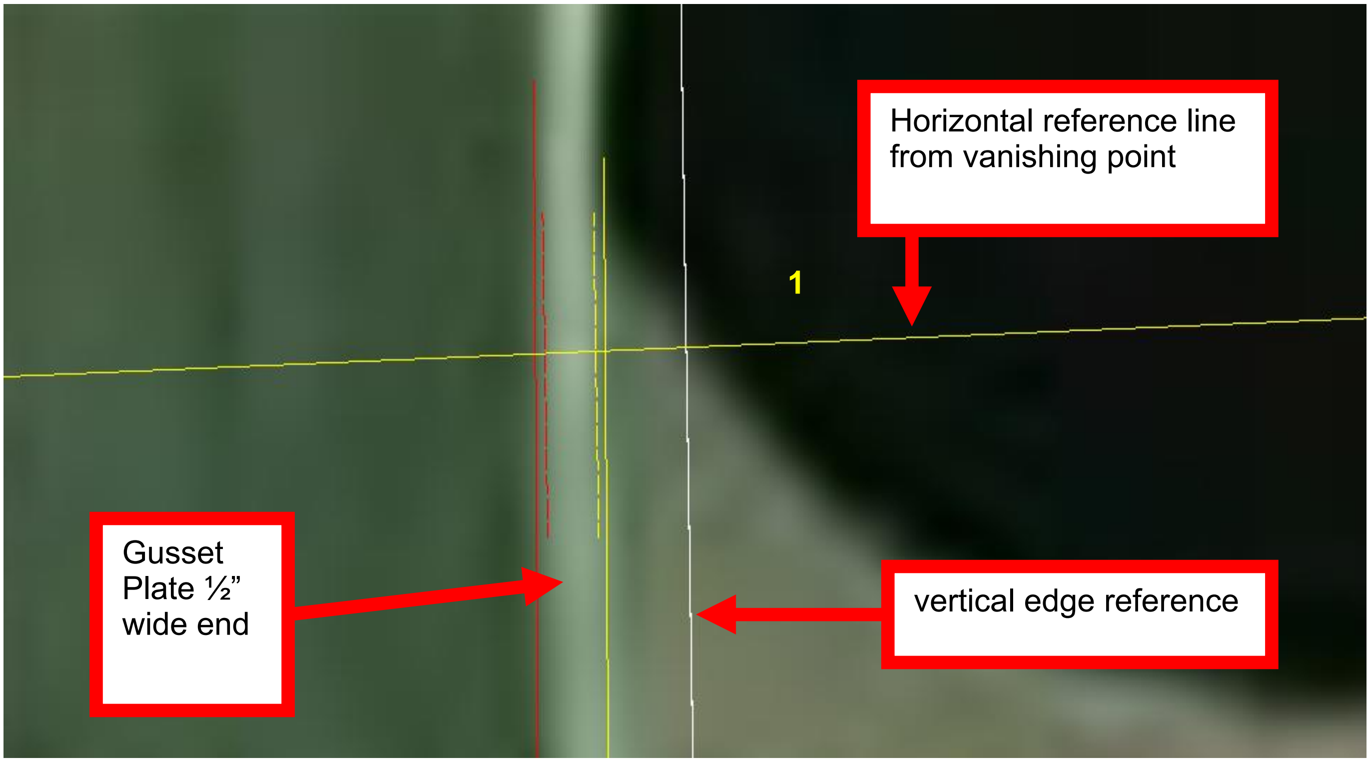

Figure 13 - Enlarged View of Bowed Gusset Plate End

The vertical red and yellow lines were drawn to mark left and right edges of the gusset plate end, respectively. The edges appear somewhat blurred in the image. As a result, two pairs of lines were constructed to bound the edge locations within a range, rather than estimating a single fixed location. The vertical white line depicts where the right edge of the gusset plate end should be, if there were no out of plane displacement. The yellow horizontal reference line was constructed from the VP, through the plate end, at the location of maximum displacement. The distance from the white vertical line to the each of the yellow vertical lines was measured. These two measurements were later used to establish the range of the gusset plate displacement.

Scale

In order to convert measurements taken directly from the photograph into displacement in 'real world' units (such as inches) a suitable scale must be established by measuring something in the photograph for which the true size is known. The item selected for this was the vertical span of the gusset plate edge, from the bottom edge of the upper chord member side plate, to the lower corner of the gusset plate.

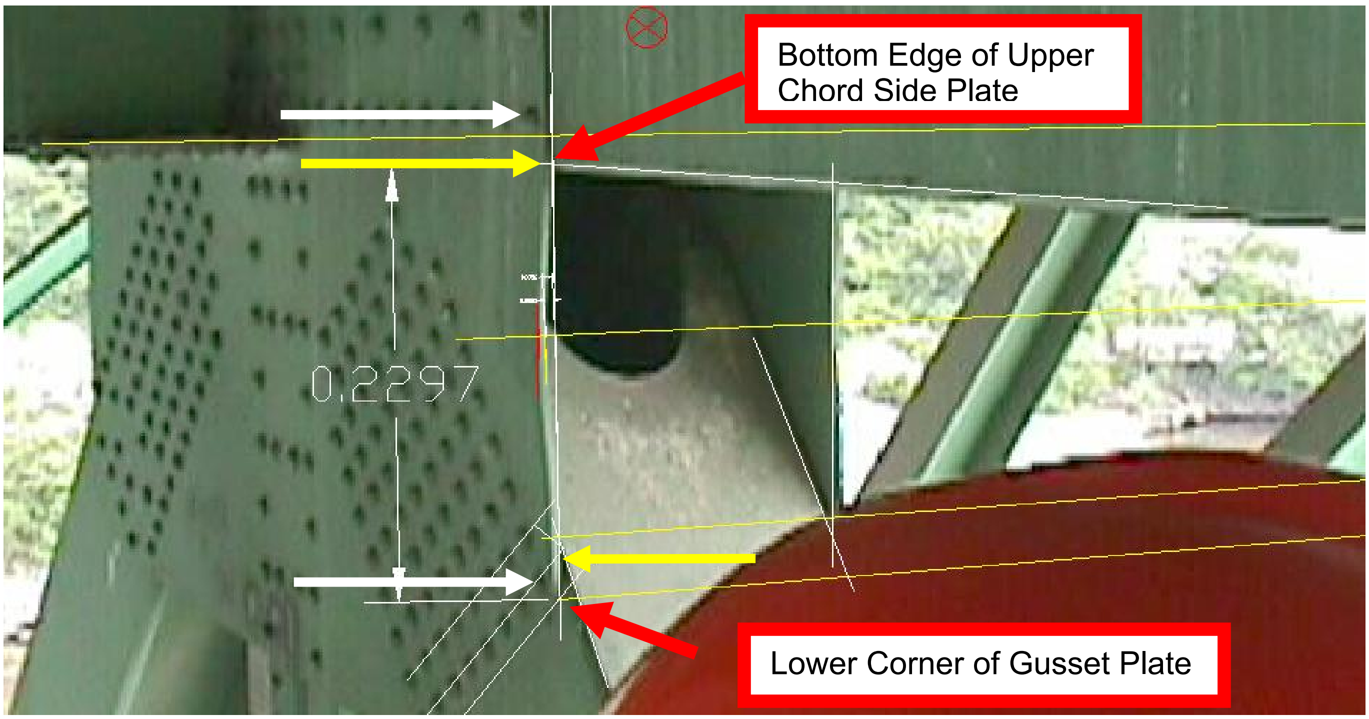

Figure 14 - Scale Object

This distance appears to be a suitable scale object considering the following:

- In the same plane as the unknown measurement of interest
- Same distance from the camera as the unknown measurement of interest
- Scale does not change significantly due to perspective
- Length of the line is relatively large (minimizing any error that may be caused by uncertainty in marking the ends of the line)

The length of this line was computed to be 28.98 inches, using shop and construction drawings for the bridge.[4]

Additionally, the length along the gusset plate edge that was unsupported (laterally) was computed from the drawings to be 29.64 inches. This assumes that the unsupported region is defined as the distance between the surfaces of the 1-inch diameter rivets denoted by the white arrows in Figure 14. An alternative definition of the unsupported length would be the region along the plate edge that is completely exposed between the upper chord side plate and the diagonal member side plate. (denoted by the yellow arrows). Using this definition, the unsupported length was computed to be 27.65 inches.

Corrections for Perspective View

The camera's view of the gusset plate edge is not orthogonal (the camera's line of sight is not perpendicular to the plate edge). As a result, the plane of the gusset plate edge is foreshortened in the image, causing the edge's thickness and any displacement to appear smaller than they actually are. To account for this, linear measurements taken from the photograph can be corrected using the following relationship:

$$D = \frac{d}{COS\theta} \quad \text{(Eqn. 1)}$$

Where:

- D is the true distance in the plane of the gusset plate edge
- d is the linear measurement taken on the photograph, and
- θ is the angle between the plane of the gusset plate edge and the camera's line of sight, (relative to orthogonal)

[4] The span between the upper chord and diagonal member - at the time the photograph was taken - is fractionally shorter than the span as computed from the design drawings, due to the bow in the gusset plate. However this difference is expected to be very small, and its effect on the on the calculations used to estimate the amount of bowing is neglected.

In order to calculate the angle from the camera to the plane of interest, a photogrammetry software program was used to orient each image, and solve for the camera's position and orientation. This was performed by computing three dimensional (x,y,z) coordinates of key features of the gusset plate and surrounding structure the shop and construction drawings, and importing those coordinates into the software. These points were then registered in each image as seen in Figure 15.

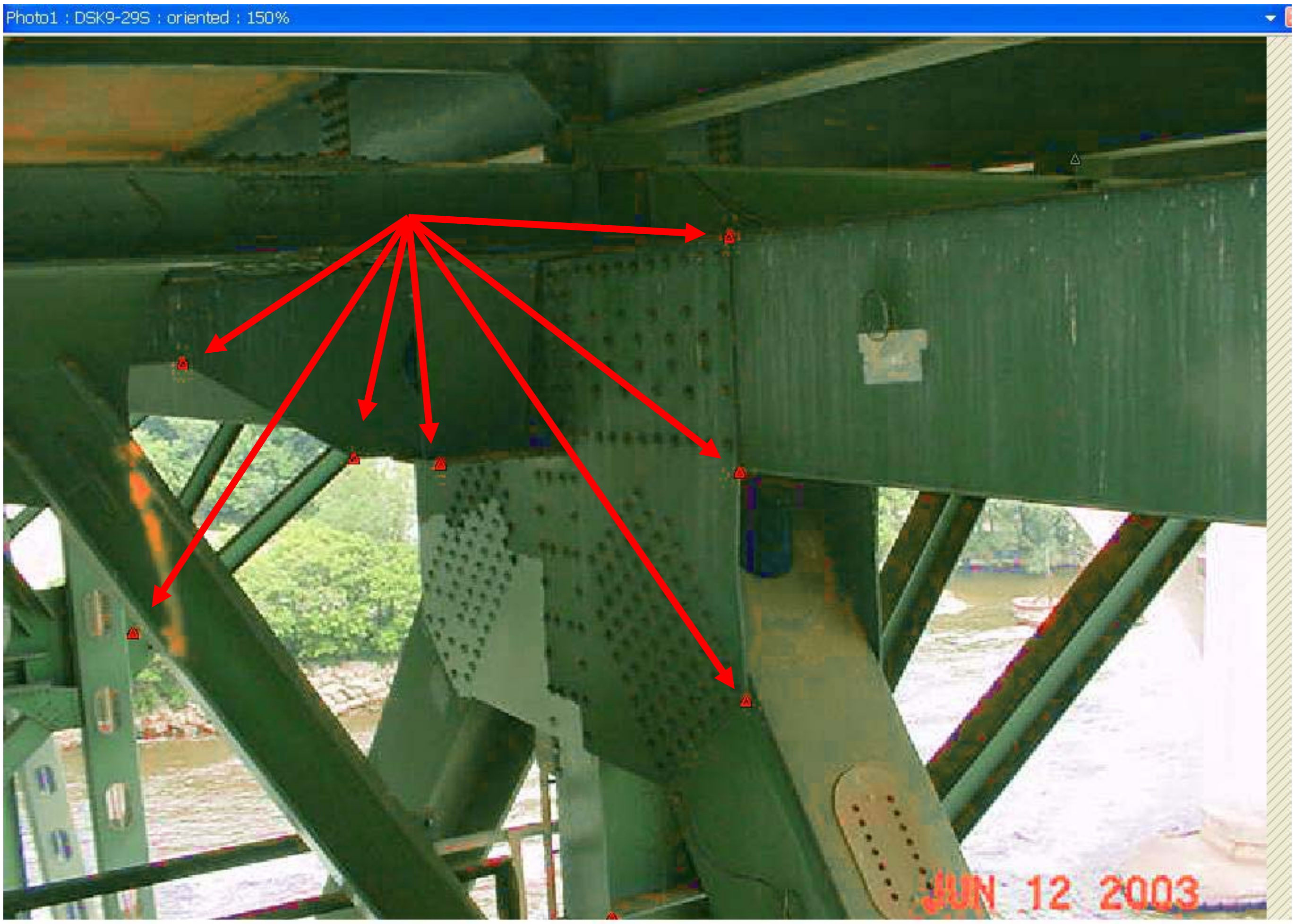

Figure 15 - Marking Coordinates in the Image[5]

[5] This figure depicts image DSK9-29S.JPG

With a properly "solved" image, the photogrammetry software can then provide the camera's location as well as it's line of sight, in the same units and coordinate system as the "real world", as defined by the set of (x,y,z) coordinates.

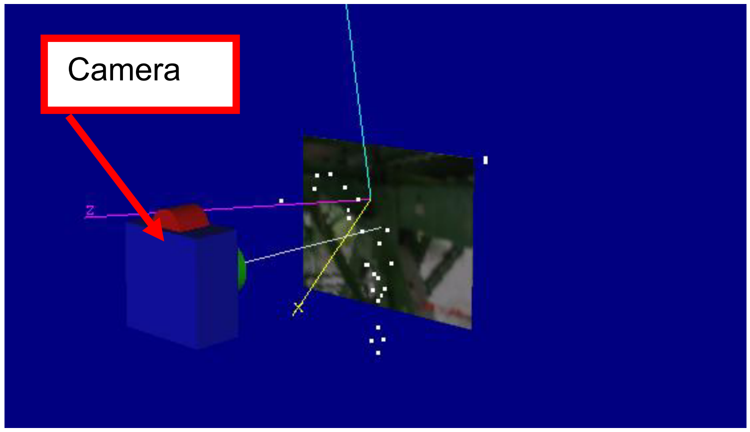

Figure 16 - Camera location

With coordinates for the camera's location, and the point of interest or "target"[6], the angle from the line of sight to an "X-Y" plane at the target on the gusset plate can be calculated as follows:

$$\theta = TAN^{-1}\left(\frac{\Delta Z}{\Delta X}\right) \qquad \text{(Eqn. 2)}$$

Where:

Θ = angle between camera line of sight and a point on the XY plane

ΔZ = (camera Z coordinate) – (target Z coordinate)

ΔX = (camera X coordinate) – (target X coordinate)

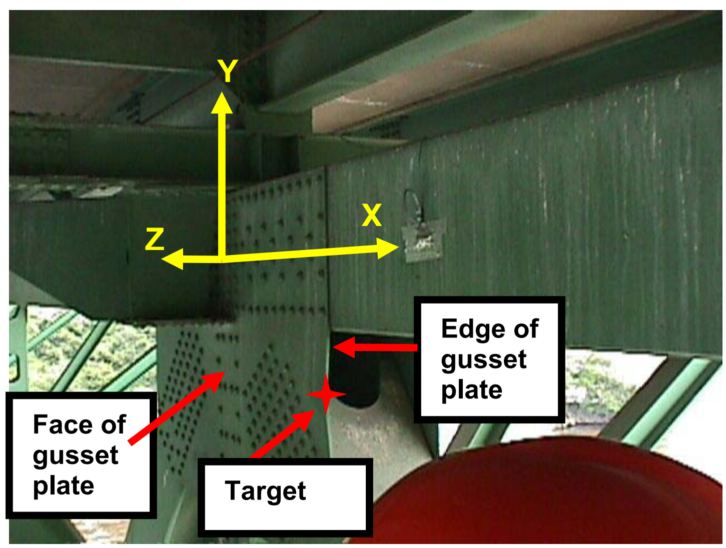

Figure 17 - Coordinate Axis

For this image, the calculated angle is about 28 degrees. This is the lateral angle between a line from the camera to the face (large side) of the gusset plate, at the plate's edge (as

[6] Here, the 'target' is a point along the span of the gusset plate edge, midway between the bottom of the upper chord side plate, and the lower corner of the gusset plate edge. The target and the camera location define the line of sight from the camera, to the area of interest. See Figure 17.

denoted by the red “star” in Figure 17) . This angle is also the deviation from a ‘straight on’, or orthogonal view of the gusset plate edge, and can be used in equation 1 noted above.

Similarly, a vertical angle between the camera’s line of sight and the edge of the gusset plate can be calculated. The span of the gusset plate between the upper chord and diagonal member (the scale distance) is below the camera location. The downward angle from the camera to the center of this span was computed to account for the slightly foreshortened view of the scale distance. This angle can be calculated as follows:

$$\phi = TAN^{-1}\left[\frac{\Delta Y}{\sqrt{(\Delta X)^2 + (\Delta Z)^2}}\right] \qquad \text{(Eqn. 3)}$$

Where:

Φ = the downward ‘elevation’ angle from the camera to an XZ plane located at the target. (The target is the midpoint of the span of the gusset plate)
ΔY = (camera Y coordinate) – (target Y coordinate)
ΔX = (camera X coordinate) – (target X coordinate)
ΔZ = (camera Z coordinate) – (target Z coordinate)

This angle for this image was calculated to be about 8 degrees.

Combining the measurements taken from the image, the established scale, and the correction factors, the out-of plane displacement of the gusset plate can be estimated.

Distance from the reference line to the right-hand side of the gusset plate edge (between vertical white line and vertical yellow lines in Figure 13: (**d**)	**0.0050 to 0.0056** (unitless)
Scale distance as measured on image: ($\mathbf{S_m}$)	**0.2297** (unitless)
Scale distance actual: ($\mathbf{S_T}$)	**28.984** inches
Lateral correction angle: (**θ**)	**28.11** degrees
Vertical correction angle: (**Φ**)	**8.29** degrees

Estimated displacement (**D**) (at the point along the plate's edge where displacement appears to be the greatest):

$$D = \frac{d\left(\frac{S_T}{S_m}\right)\left(\frac{1}{COS(\phi)}\right)}{COS(\theta)} \qquad \text{(Eqn. 4)}$$

Solving equation 4 using both the minimum and maximum values of **d** yields a displacement of the gusset plate of 0.72 to 0.81 inches.

Dimensional Ratio Method

Another way to estimate the gusset plate displacement and account for the effects of the perspective view is to compare a ratio of two known actual dimensions to their relationship as seen in the perspective view. Consider the following example.

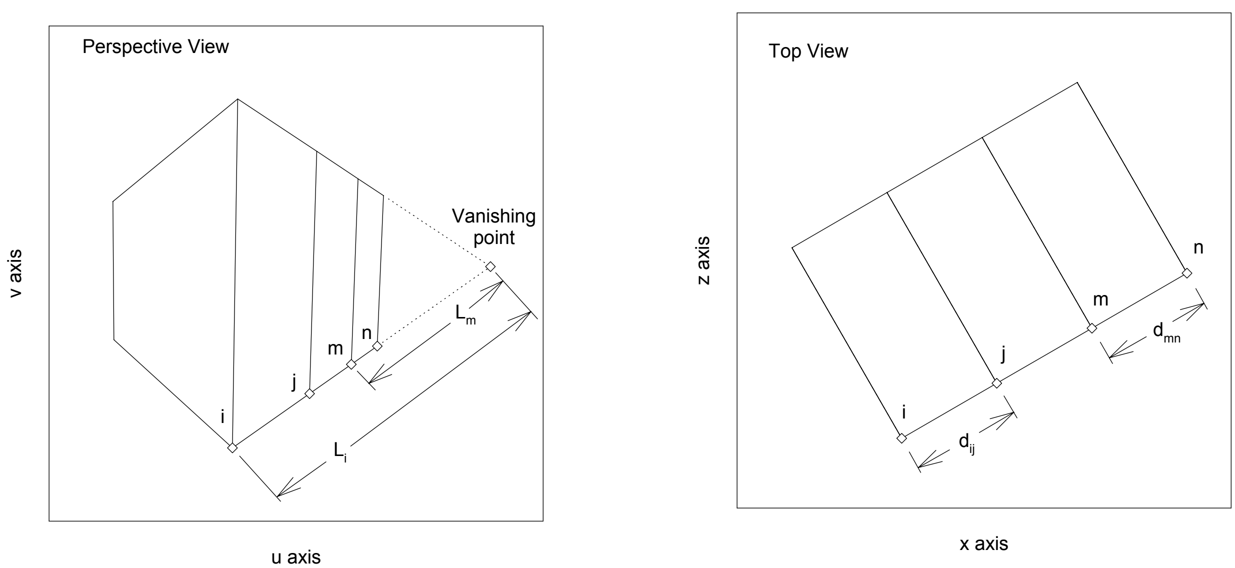

Figure 18 – Perspective and Top Views

The left-hand view in Figure 18 shows a perspective view of a rectangular solid with 4 equally spaced lines drawn on the side. The lengths along the side are foreshortened because they are viewed at an angle, and lengths farther away are additionally reduced by the perspective projection. Parallel lines can be extended to a vanishing point. Four points along the bottom edge (i, j, m and n) are indicated, along with two lengths measured from the vanishing point, L_i and L_m. The right-hand view in Figure 18 shows a top view of the rectangular solid, with the same points indicated, along with true distances d_{ij} and d_{mn}. This view of the top of the solid is orthogonal, and lengths depicted are not foreshortened.

In this example, the ratio of two actual dimensions (d) from the right-hand view can be expressed as a ratio of lengths from the points along a straight line to the vanishing point, in the left-hand view. For example:

$$\frac{d_{ij}}{d_{mn}} = \frac{L_m L_n (L_j - L_i)}{L_i L_j (L_n - L_m)}$$ (demonstrative example equation)

More information on the derivation of this relationship can be found in Attachment II.

A similar approach can be applied to the images of the gusset plates.

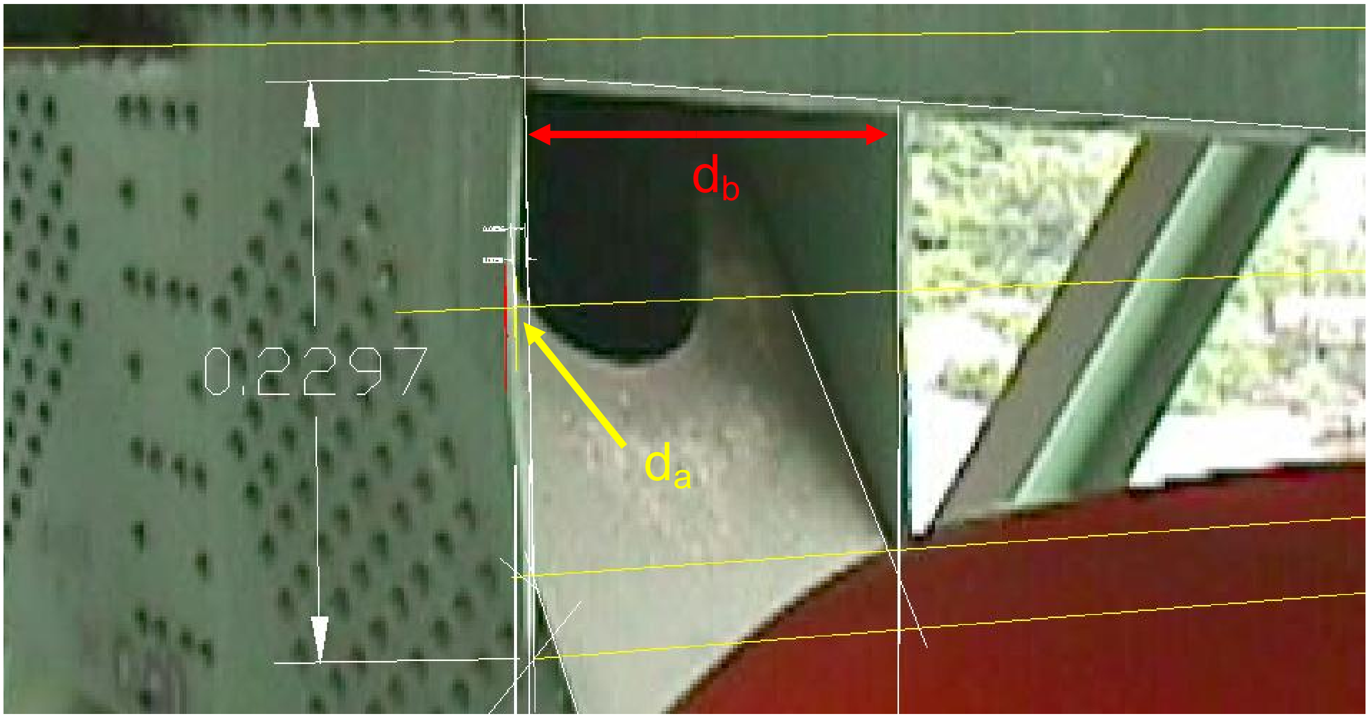

Figure 19 – Dimensional Ratio Method

The distance across the extents of the diagonal member (the distance between the inner sides of two unbowed gusset plates) noted as d_b is 21.0 inches as computed from the shop and construction drawings. The distance d_a indicated by the yellow arrow, is the unknown displacement of the inside corner of the gusset plate edge.

Similar to the example above, the ratio of d_a to d_b can be expressed as a function of linear distances measured from the points of interest to a vanishing point for the image.

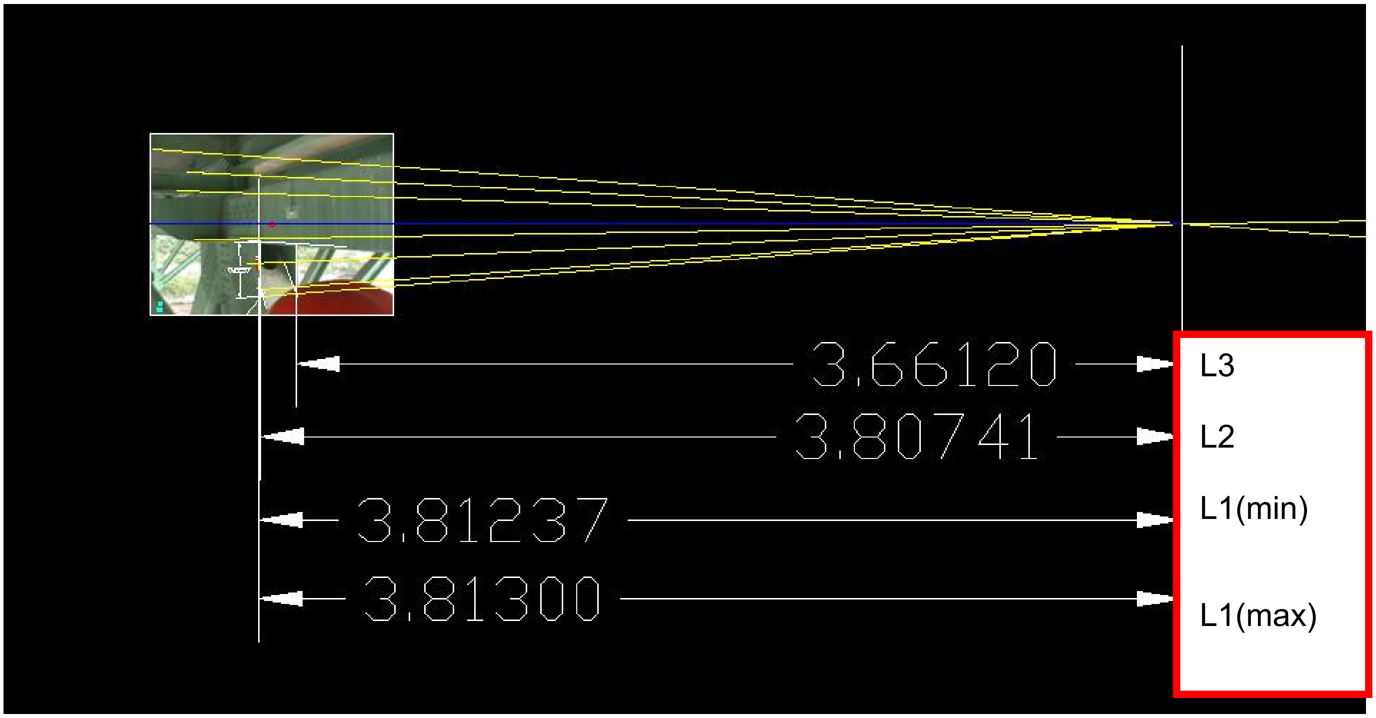

Figure 20 - Measurements to Vanishing Point

In this case, the relationship is

$$\frac{d_a}{d_b} = \frac{L_3(L_1 - L_2)}{L_1(L_2 - L_3)} \qquad \text{(Eqn. 5)}$$

Where:

d_a = gusset plate displacement in inches

d_b = diagonal beam width (21.0 inches)

L_1 = distance measured in image from the right side of gusset plate edge (the inside corner) to the vanishing point.[7]

L_2 = distance measured in image from the vertical line representing the right side of the gusset plate edge (the inside corner), if the plate were flat.

[7] Again, this measurement was quantified with a minimum and a maximum due to the uncertainty in marking the edge of the plate in image.

L_3 = distance measured in image from the left side of the opposite gusset plate edge (the inside corner) to the vanishing point.

Solving equation 5 using both the minimum and maximum values of L_1 yields a displacement of the gusset plate of 0.69 to 0.77 inches.

Summary of Results

Both of the methodologies described above were applied to the other image of the U10 East node (image DSK9-029S.jpg), the 3 images of U10 West node (DSK5-02S.JPG, DSK5-03S.JPG, DSK5-04S.JPG), one image of the U10′ West node (DSK4-04S.JPG) and one image of the U10′ East node (DSK9-02S.JPG). The results are presented in Table 1.

Image name	Date of Image	Node	Gusset	Calculated Displacement (largest along plate edge) Inches			
				Direct Scale Method		Dimensional Ratio Method	
				min	max	min	max
MVC-010F.JPG	8/10/1999	U10E	west (inner)	0.72	0.81	0.69	0.77
DSK9-029S.JPG	6/12/2003	U10E	west (inner)	0.73	0.81	0.70	0.77
DSK5-02S.JPG	6/10/2003	U10W	east(inner)	0.56	0.64	0.52	0.59
DSK5-03S.JPG	6/10/2003	U10W	east(inner)	0.47	0.54	0.50	0.57
DSK5-04S.JPG	6/10/2003	U10W	east(inner)	0.48	0.55	0.46	0.52
DSK9-02S.JPG	6/12/2003	U10’ E	west (inner)	0.53	0.60	0.49	0.56
DSK4-04S.JPG	6/10/2003	U10’ W	east (inner)	0.82	0.99	0.84	0.99

Table 1 - Calculated Gusset Plate Displacements

Other U10 / U10′ Displaced Gusset Plate Ends

Although not all of the gusset plate ends could be measured for displacement, all of the images of the U10 and U10′ nodes found in the "Pre-Collapse Images – Nodes U10 and L11" document were reviewed for any indication of bowing in gusset plate ends. The following observations were made:

Node	Plate	End of plate	Apparent displacement Y/N	Direction of Displacement
U10 East	Outer	Southern	Yes	West
U10 East	Inner	Southern	Yes (measured)	West
U10 East	Outer	Northern	No	
U10 East	Inner	Northern	No	
U10 West	Outer	Southern	Yes	West
U10 West	Inner	Southern	Yes (measured)	West
U10 West	Outer	Northern	No	
U10 West	Inner	Northern	Unable to determine	
U10′ East	Outer	Southern	[No photos]	
U10′ East	Inner	Southern	[No photos]	
U10′ East	Outer	Northern	Unable to determine	
U10′ East	Inner	Northern	Yes(measured)	East
U10′ West	Outer	Southern	No	
U10′ West	Inner	Southern	No	
U10′ West	Outer	Northern	Yes	East
U10′ West	Inner	Northern	Yes(measured)	East

Doug Brazy
Mechanical Engineer - CVR/Video
NTSB Vehicle Recorder Division

Attachment I

Measurements and numerical data

Camera Stations		(these values were computed by the photogrammetry software package)					
		Camera Location			**Point Along Camera's Line of Sight**		
Image name	**Bridge Node**	**Center X (in)**	**Center Y (in)**	**Center Z (in)**	**Target X (in)**	**Target Y (in)**	**Target Z (in)**
DSK5-02S.jpg	U10 W	-191.2984278	-31.46811429	78.62208141	-4.812987591	-26.60585286	9.774326791
MVC-010F.JPG	U10 E	169.2480201	-8.500403289	64.13414484	58.84799764	-5.827655831	-1.659786831
DSK9-029S.JPG	U10E	202.8725685	4.210472021	103.6742564	-20.70077528	-28.43275747	-21.07003242
DSK5-03S.JPG	U10W	-242.1547093	-11.47030673	125.0020271	11.35327199	-24.02998052	-4.855099767
DSK5-04S.JPG	U10W	-237.0036472	-12.27589255	124.5790619	6.064659676	-24.63422281	-1.182953036
DSK9-02S.JPG	U10Prime E	-202.8538618	14.81534386	150.642519	4.61025661	-21.97272428	-25.38539593
DSK4-04S.JPG	U10Prime W	299.8710177	-62.00021898	158.7528643	10.0500905	-29.09390892	-1.49066319
Below, X is negated for the East nodes, positive Z is inward toward the bridge centerline							
"GP_MID" is a known point at mid span of the inner gusset plate. Along with the Camera Location coordinate(s) above, is used to define a vector from the camera location to the area of interest (the location of maximum displacement of the gusset plate edge).							
	X (in)	**Y (in)**	**Z (in)**				
GP_MID (east nodes)	48.25	-28.492	-0.5				
GP_MID (west nodes)	-48.25	-28.492	-0.5				

Direct Scaling Method

DSK9-029S.JPG						
Camera Axis angle to the XY plane = TAN^{-1} [ΔZ/ΔX] (using camera **location** point, to camera **target** point)						29.15963387
"Lateral Angle" = Camera Center to point GPMID angle to the XY plane = TAN^{-1} [ΔZ/ΔX] (using camera **Location** point to the "**GP_MID**" point						33.96943151
"Vertical Angle" from camera **location** point an XZplane at **GP_MID** point			$TAN^{-1}\left[\frac{\Delta Y}{\sqrt{\left(\Delta X^2 + \Delta Z^2\right)}}\right]$			9.948672589
Image measured Displacement min			0.0036			
Image measured Displacement max			0.004			
scale item measured (measure from point GP10 - +0.5 in Y, to GP8)				0.1743		
scale item true distance		28.984	inches			
scale 'factor' = $\frac{ScaleItemTrue}{ScaleItemMeasured} \times \frac{1}{COS(VerticalAngle)}$				169.8106287		
min displacement	**=** $\frac{MeasuredDisplacementMin \times ScaleFactor}{COS(LateralAngle)}$			**0.732846712**	**inches**	
max displacement	**=** $\frac{MeasuredDisplacementMax \times ScaleFactor}{COS(LateralAngle)}$			**0.814274125**	**inches**	

MVC-010F.JPG						
Camera Axis angle to the XY plane = TAN^{-1} [ΔZ/ΔX] (using camera **location** point, to camera **target** point)						30.79322581
"Lateral Angle" = Camera Center to point GPMID angle to the XY plane = TAN^{-1} [ΔZ/ΔX] (using camera **Location** point to the "**GP_MID**" point						28.11002853
"Vertical Angle" from camera **location** point an XZplane at **GP_MID** point			$TAN^{-1}\left[\frac{\Delta Y}{\sqrt{(\Delta X^2 + \Delta Z^2)}}\right]$			8.291549165
Image measured Displacement min			0.005			
Image measured Displacement max			0.0056			
scale item measured (measure from point GP10 - +0.5 in Y, to GP8)				0.2297		
scale item true distance		28.984	inches			
scale 'factor' = $\frac{ScaleItemTrue}{ScaleItemMeasured} \times \frac{1}{COS(VerticalAngle)}$				127.847506		
min displacement	= $\frac{MeasuredDisplacementMin \times ScaleFactor}{COS(LateralAngle)}$			**0.722837076**	**inches**	
max displacement	= $\frac{MeasuredDisplacementMax \times ScaleFactor}{COS(LateralAngle)}$			**0.809577525**	**inches**	

DSK5-02S.JPG						
Camera Axis angle to the XY plane = TAN^{-1} [ΔZ/ΔX] (using camera **location** point, to camera **target** point)						-20.26342412
"Lateral Angle" = Camera Center to point GPMID angle to the XY plane = TAN-1 [ΔZ/ΔX] (using camera **Location** point to the **"GP_MID" point**						-28.94759355
"Vertical Angle" from camera **location** point an XZplane at **GP_MID** point			$TAN^{-1}\left[\frac{\Delta Y}{\sqrt{(\Delta X^2+\Delta Z^2)}}\right]$			-1.042990579
Image measured Displacement min			0.0035			
Image measured Displacement max			0.004			
scale item measured (measure from point GP10 - +0.5 in Y, to GP8)				0.2074		
scale item true distance		28.984	inches			
scale 'factor' = $\frac{ScaleItemTrue}{ScaleItemMeasured}\times\frac{1}{COS(VerticalAngle)}$				139.7795053		
min displacement	= $\frac{MeasuredDisplacementMin \times ScaleFactor}{COS(LateralAngle)}$			**0.559049627**	**inches**	
max displacement	= $\frac{MeasuredDisplacementMax \times ScaleFactor}{COS(LateralAngle)}$			**0.638913859**	**inches**	

DSK5-03S.JPG						
Camera Axis angle to the XY plane = TAN^{-1} [ΔZ/ΔX] (using camera **location** point, to camera **target** point)						-27.12337585
"Lateral Angle" = Camera Center to point GPMID angle to the XY plane = TAN-1 [ΔZ/ΔX] (using camera **Location** point to the **"GP_MID" point**						-32.91238095
"Vertical Angle" from camera **location** point an XZplane at **GP_MID** point			$TAN^{-1}\left[\frac{\Delta Y}{\sqrt{\left(\Delta X^2 + \Delta Z^2\right)}}\right]$			4.21477751
Image measured Displacement min			0.0025			
Image measured Displacement max			0.0029			
scale item measured (measure from point GP10 - +0.5 in Y, to GP8)				0.1856		
scale item true distance		28.984	inches			
scale 'factor' = $\frac{ScaleItemTrue}{ScaleItemMeasured} \times \frac{1}{COS(VerticalAngle)}$				156.7597535		
min displacement	**=** $\frac{MeasuredDisplacementMin \times ScaleFactor}{COS(LateralAngle)}$			**0.466309749**	**inches**	
max displacement	**=** $\frac{MeasuredDisplacementMax \times ScaleFactor}{COS(LateralAngle)}$			**0.540919309**	**inches**	

DSK5-04S.JPG						
Camera Axis angle to the XY plane = TAN^{-1} [ΔZ/ΔX] (using camera **location** point, to camera **target** point)						-27.35676142
"Lateral Angle" = Camera Center to point GPMID angle to the XY plane = TAN-1 [ΔZ/ΔX] (using camera **Location** point to the **"GP_MID" point**						-33.53075314
"Vertical Angle" from camera **location** point an XZplane at **GP_MID** point			$TAN^{-1}\left[\frac{\Delta Y}{\sqrt{\left(\Delta X^{2}+\Delta Z^{2}\right)}}\right]$			4.096239066
Image measured Displacement min			0.0031			
Image measured Displacement max			0.0035			
scale item measured (measure from point GP10 - +0.5 in Y, to GP8)				0.2238		
scale item true distance		28.984	inches			
scale 'factor' = true/measured * 1/cos(vertical angle)				129.9820593		
min displacement	$=\frac{MeasuredDisplacementMin \times ScaleFactor}{COS(LateralAngle)}$			**0.482857029**	**inches**	
max displacement	$=\frac{MeasuredDisplacementMax \times ScaleFactor}{COS(LateralAngle)}$			**0.545161161**	**inches**	

DSK9-02S.JPG						
Camera Axis angle to the XY plane = TAN^{-1} [ΔZ/ΔX] (using camera **location** point, to camera **target** point)						-40.3137395
"Lateral Angle" = Camera Center to point GPMID angle to the XY plane = TAN-1 [ΔZ/ΔX] (using camera **Location** point to the **"GP_MID" point**						-44.35138444
"Vertical Angle" from camera **location** point an XZplane at **GP_MID** point			$TAN^{-1}\left[\frac{\Delta Y}{\sqrt{(\Delta X^2 + \Delta Z^2)}}\right]$			11.32662284
Image measured Displacement min			0.0028			
Image measured Displacement max			0.0032			
scale item measured (measure from point GP10 - +0.5 in Y, to GP8)				0.2190		
scale item true distance		28.984	inches			
scale 'factor' = true/measured * 1/cos(vertical angle)				134.9758892		
min displacement	$= \frac{MeasuredDisplacementMin \times ScaleFactor}{COS(LateralAngle)}$			**0.528528057**	**inches**	
max displacement	$= \frac{MeasuredDisplacementMax \times ScaleFactor}{COS(LateralAngle)}$			**0.604032065**	**inches**	

DSK4-04S.JPG						
Camera Axis angle to the XY plane = TAN^{-1} [ΔZ/ΔX] (using camera **location** point, to camera **target** point)						28.93843853
"Lateral Angle" = Camera Center to point GPMID angle to the XY plane = TAN-1 [ΔZ/ΔX] (using camera **Location** point to the **"GP_MID" point**						32.33003219
"Vertical Angle" from camera **location** point an XZplane at **GP_MID** point			$TAN^{-1}\left[\frac{\Delta Y}{\sqrt{(\Delta X^2 + \Delta Z^2)}}\right]$			-6.420240455
Image measured Displacement min			0.0049			
Image measured Displacement max			0.0059			
scale item measured (measure from point GP10 - +0.5 in Y, to GP8)				0.2053		
scale item true distance		28.984	inches			
scale 'factor' = true/measured * 1/cos(vertical angle)				134.9758892		
min displacement	$= \frac{MeasuredDisplacementMin \times ScaleFactor}{COS(LateralAngle)}$			**0.823854366**	**inches**	
max displacement	$= \frac{MeasuredDisplacementMax \times ScaleFactor}{COS(LateralAngle)}$			**0.99198791**	**inches**	

Dimensional Ratio Method

							Displacement d_a	
DSk9-029S.JPG				Using L1(min)		Using L1(max)	**Min Inches**	**Max-Inches**
$\frac{d_a}{d_b} = \frac{L3 \times (L1 - L2)}{L1 \times (L2 - L3)}$			$\frac{d_a}{d_b} =$	0.008501 / 0.266596	$\frac{d_a}{d_b} =$	0.009458 / 0.266596	**0.695914**	**0.774951**
da	member overall width			21	inches			
db	Plate edge to reference line			?				
Measurements								
L3	member top face edge to VP			2.31862				
L1	Plate Edge to VP			2.42633(min)	2.42674(max)			
L2	reference line to VP			2.42272				
autocad_measure_04_21_08.dwg								

							Displacement d_a	
MVC-010F.JPG				Using L1(min)		Using L1(max)	**Min Inches**	**Max-Inches**
$\frac{d_a}{d_b} = \frac{L3 \times (L1 - L2)}{L1 \times (L2 - L3)}$			$\frac{d_a}{d_b} =$	0.01816 / 0.557407	$\frac{d_a}{d_b} =$	0.020466 / 0.557407	**0.686152**	**0.773305**
da	member overall width			21	Inches			
db	Plate edge to reference line			?				
Measurements								
L3	member top face edge to VP			3.66161				
L1	Plate Edge to VP			3.81237(min)	3.813(max)			
L2	reference line to VP			3.80741				
note: hard to determine L3 with helmet in the way								

							Displacement $\mathbf{d_a}$	
DSK5-02S.JPG				Using L1(min)		Using L1(max)	**Min Inches**	**Max- Inches**
$\frac{d_a}{d_b} = \frac{L3 \times (L1 - L2)}{L1 \times (L2 - L3)}$			$\frac{d_a}{d_b} =$	-0.01437 / 0.581722	$\frac{d_a}{d_b} =$	-0.01634 / 0.581722	**-0.51863**	**-0.58996**
db	member overall width			21	inches			
da	Plate edge to reference line			?				
Measurements								
L3	member top face edge to VP			4.11648				
L1	Plate Edge to VP			4.24987(min)	4.24939(max)			
L2	Reference line to VP			4.25336				
all_images_u10_west_04_25_08.dwg								
							Displacement $\mathbf{d_a}$	
DSK5-03S.JPG				Using L1(min)		Using L1(max)	**Min Inches**	**Max- Inches**
$\frac{d_a}{d_b} = \frac{L3 \times (L1 - L2)}{L1 \times (L2 - L3)}$			$\frac{d_a}{d_b} =$	-0.00821 / 0.344955	$\frac{d_a}{d_b} =$	-0.00935 / 0.344955	**-0.49971**	**-0.56932**
db	member overall width			21	inches			
da	Plate edge to reference line			?				
Measurements								
L3	member top face edge to VP			2.93161				
L1	Plate Edge to VP			3.0422(min)	3.04181(max)			
L2	reference line to VP			3.045				

							Displacement d_a	
DSK5-04S.JPG				Using L1(min)		Using L1(max)	**Min Inches**	**Max- Inches**
$\frac{d_a}{d_b} = \frac{L3 \times (L1 - L2)}{L1 \times (L2 - L3)}$			$\frac{d_a}{d_b} =$	-0.01498 0.67922	$\frac{d_a}{d_b} =$	-0.01694 0.67922	**-0.46312**	**-0.52366**
Db	member overall width			21	inches			
Da	Plate edge to reference line			?				
Measurements								
L3	member top face edge to VP			4.89511				
L1	Plate Edge to VP			5.02716(min)	5.02676(max)			
L2	reference line to VP			5.03022				

							Displacement d_a	
DSK4-04S.JPG				Using L1(min)		Using L1(max)	**Min Inches**	**Max- Inches**
$\frac{d_a}{d_b} = \frac{L3 \times (L1 - L2)}{L1 \times (L2 - L3)}$			$\frac{d_a}{d_b} =$	0.01859 0.467487	$\frac{d_a}{d_b} =$	0.021996 0.467487	**0.83507**	**0.988084**
Db	member overall width			21	inches			
Da	Plate edge to reference line			?				
Measurements								
L3	member top face edge to VP			3.62373				
L1	Plate Edge to VP			3.75341(min)	3.75435(max)			
L2	reference line to VP			3.74828				

							Displacement d_a	
DSK9-02S.JPG				Using L1(min)		Using L1(max)	**Min Inches**	**Max-Inches**
$\frac{d_a}{d_b} = \frac{L3 \times (L1 - L2)}{L1 \times (L2 - L3)}$			$\frac{d_a}{d_b} =$	-0.00553	$\frac{d_a}{d_b} =$	-0.00634	**-0.48769**	**-0.55915**
				0.238092		0.238092		
Db	member overall width			21	inches			
Da	Plate edge to reference line			?				
Measurements								
L3	member top face edge to VP			2.02538				
L1	Plate Edge to VP			2.13421(min)	2.13381(max)			
L2	reference line to VP			2.13694				

Attachment II

Dimensional Ratio Method Derivation

(Provided by Carl Schultheisz, NTSB Materials Laboratory Division)

Perspective projection

Assume there is an observer at $(0,0,0)$ and a screen parallel to the $x-y$ plane at $z=z_0$. Let u denote the coordinate in the plane of the screen parallel to the x axis, and let v denote the coordinate in the plane of the screen parallel to the y axis. Then a point (x,y,z) will be projected to a position (u,v,z_0) on the screen given by

$$\begin{pmatrix} u \\ v \\ z_0 \end{pmatrix} = \frac{z_0}{z}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

A straight line can be parameterized as

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} + t\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Parallel lines would have the same b_i's but different a_i's. The perspective projection of a straight line is also a straight line, with

$$\begin{pmatrix} u \\ v \\ z_0 \end{pmatrix} = \frac{z_0}{z}\left[\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} + t\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}\right] = \frac{z_0}{a_3+tb_3}\left[\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} + t\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}\right]$$

Perspective projections of parallel lines (other than those with z = constant) will meet at a vanishing point (at $t \to \infty$), given by

$$\begin{pmatrix} u \\ v \\ z_0 \end{pmatrix}_{VP} = \frac{z_0}{b_3}\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

The distance from a point along the perspective projection of a line to its vanishing point depends on the quantity

$$\begin{pmatrix} u \\ v \\ z_0 \end{pmatrix} - \begin{pmatrix} u \\ v \\ z_0 \end{pmatrix}_{VP} = \frac{z_0}{a_3+tb_3}\left[\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} + t\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}\right] - \frac{z_0}{b_3}\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$$= \frac{z_0}{a_3+tb_3}\left[\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} + \frac{a_3}{b_3}\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}\right]$$

The quantities inside the square brackets are constant for a specific line, as is the screen position z_0. Calculating the distance would require squaring the scalars and taking the dot product of the vectors, and then taking the square root of the resultant. The net result would show that the distance from a point along the perspective projection of a line to its vanishing point (call that L) is given by

$$L = \frac{k}{a_3+tb_3} = \frac{k}{z}$$

where k is a constant for a given line. The distance L is therefore inversely proportional to the z coordinate of the point. A similar relation would hold for each component u and v, with

$$u - u_{VP} = \frac{z_0}{a_3 + tb_3}\left[a_1 - \frac{a_3}{b_3}b_1\right] = \frac{k_u}{z}$$

$$v - v_{VP} = \frac{z_0}{a_3 + tb_3}\left[a_2 - \frac{a_3}{b_3}b_2\right] = \frac{k_v}{z}$$

where k_u and k_v are constants for a given line as defined in the preceding equations.

The distance d_{ij} between two points i and j along a line depends on the quantity

$$\begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix} - \begin{pmatrix} x_j \\ y_j \\ z_j \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} + t_i \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} - \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} - t_j \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$$= (t_i - t_j)\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

The distance d_{ij} is then given by

$$d_{ij} = (t_i - t_j)\left[b_1^{\,2} + b_2^{\,2} + b_3^{\,2}\right]^{1/2}$$
$$= c(t_i - t_j)$$

where c is a constant for any line. All three coordinates are parameterized by t, including $z = a_3 + tb_3$, so

$$z_i - z_j = b_3(t_i - t_j)$$

The distance d_{ij} along the line can therefore be written as

$$d_{ij} = \hat{c}(z_i - z_j)$$

where $\hat{c}$ is also constant for any line.

If four points are considered, i, j, m and n, ratios of distances can be used to remove the constants, with

$$\frac{d_{ij}}{d_{mn}} = \frac{z_i - z_j}{z_m - z_n}$$

The ratios of the distances along the line can then be related to the length from the perspective projection of each point (i, j, m and n) to the vanishing point.

$$\frac{d_{ij}}{d_{mn}} = \frac{L_m L_n (L_j - L_i)}{L_i L_j (L_n - L_m)}$$

Thus, if one true distance d_{ij} is known, and the L_i's can be measured, another distance d_{mn} can be calculated.