Docket No. SA-509

Exhibit No. 5-G

NATIONAL TRANSPORTATION SAFETY BOARD

Washington D.C.

Calculations of Liquid Water Content, Rainfall Rate, and Percent of Water by Weight of Air

National Transportation Safety Board
Office of Aviation Safety
Washington D.C. 20594

September 2, 1994

To: John Young AS-40

From: Gregory Salottolo AS-30

Subject: Calculations of Liquid Water Content, Rainfall Rate, and Percent of Water by Weight of Air. (DCA-94-MA-065)

The Liquid Water Content and Rainfall Rate encountered by USAir 1016 prior to impact can not be accurately determined. However, an estimate of the Liquid Water Content and Rainfall Rate will be made using radar reflectivity data from the WSR-88D Doppler Weather Radar located at Columbia, South Carolina (about 77 nautical miles to the south). The WSR-88D data showed a weather echo reflectivity of about 55 DBZ at 2241Z near the initial impact point of the airplane; the maximum reflectivity value noted in this echo. Because an exact value was not available in the data the actual value of reflectivity could reside in the interval 55 to <60 DBZ inclusive. All values from 55 to <60 DBZ would be displayed as 55 DBZ. The data from the .5 degree elevation scan was used. At this elevation angle the beam center was at about 8,400 feet above mean sea level (msl) and the beam width about 7,800 feet. The equation relating Liquid Water Content to weather radar reflectivity was obtained from a paper by Greene and Clark on Vertically Integrated Liquid Water. See Attachment 1. The equation is as follows:

$M = 3.44X10^{-3}Z^{4/7}$ where M = Liquid Water Content in grams per cubic meter and Z = weather radar reflectivity in millimeters to the sixth power (mm^6) divided by meters cubed (m^3).

$$DBZ = 10 \times LOG(Z)$$
$$Z = 10^{DBZ/10}$$

The following results were obtained by entering the following values of DBZ into the above equation:

DBZ	Liquid Water Content (grams per cubic meter)
55	4.78

(1)

56	5.45*
57	6.22*
58	7.09*
59	8.09*
60	9.23*

* According to the National Weather Service caution must be exercised in using values obtained from reflectivities above 55 DBZ due to the possible affects of hail on the radar reflectivity.

Rainfall rates were also calculated using the equation $R = (Z/300)^{.714}$ where R = rainfall rate in millimeters per hour and Z = weather radar reflectivity. The equation was obtained from Federal Meteorological Handbook No. 11 Part B page 5-6 equation (5-16). The equation noted in the FMH was solved for R. See Attachment 2.

Given the following values of DBZ the following values of R converted to inches per hour were calculated:

DBZ	Rainfall Rate (inches per hour)
53	4.08
55	5.67*
56	6.68*
57	7.88*
58	9.28*
59	10.94*
60	12.90*

* According to the National Weather Service caution must be exercised in using values obtained from reflectivities above 53 DBZ because of the possible affects of hail on the radar reflectivity.

It should be noted that extreme values of Liquid Water Content and Rainfall Rate over a small time period can be an order of magnitude greater than the above calculated values (Extremes of Precipitation Rate and Water Concentrations for the Worldwide Air Environment, Proprietary Information, The Boeing Company).

Calculations of the percent of water by weight to air were made using an air density of 1140 grams per cubic meter and the previously calculated values of Liquid Water Content. The density was calculated using a temperature of 25 degrees C, dew point temperature of 25 degrees C, and a pressure of 988.83 millibars; estimated conditions in the area of the accident. The following Table relates Liquid Water Content and percent by weight of water to air:

Liquid Water Content (grams per cubic meter)	Water % by Weight
4.78	.419
5.45	.478
6.22	.546
7.09	.622
8.09	.710
9.23	.810

Attachments: 1) Paper on Vertically Integrated Liquid Water
2) Excerpt FMH 11.

[redacted]

Gregory D. Salottolo
National Resource Specialist
Meteorology

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Vertically Integrated Liquid Water—A New Analysis Tool

DOUGLAS R. GREENE—*Techniques Development Laboratory, NOAA, Silver Spring, Md.*
ROBERT A. CLARK—*Department of Meteorology, Texas A&M University, College Station, Tex.*

ABSTRACT—Through the use of digital radar data measured at successive elevation angles in a storm system, we developed a technique that presents a new dimension in mesoscale analysis. This technique, mapped vertically integrated liquid-water content (VIL), presents the three-dimensional characteristics of a storm system in a two-dimensional display. This analysis technique appears to hold real promise for both severe storm and hydrologic applications.

1. INTRODUCTION

The space-time density of regular synoptic meteorological data is inadequate for many meteorological purposes. This data gap is partially filled by weather radars that can effectively scan a radius of 100 mi and, in some cases, an entire storm-producing area. In most cases, these radar data are reduced manually by field forecasters for short-range forecasting and issuing of storm warnings (Bigler et al. 1970). Manual techniques are frequently impractical because (1) the large quantities of data generated by the radar are too difficult to assimilate and (2) the visual extrapolation of radar data is often unreliable because of rapid changes in small-scale echo characteristics. The ultimate solution is "real-time" automatic computer processing and analysis of radar data. Steps were taken toward a real-time system with the development of the digitizing hardware and procedures for the WSR–57 by the National Severe Storms Laboratory (NSSL) of the National Oceanic and Atmospheric Administration (NOAA) and by the initiation in spring 1971 of an experiment in digitizing weather radar data from a four-station network by the National Weather Service (NWS), NOAA. Digital radar data have been used for hydrologic applications and by Barclay and Wilk (1970) to identify and track storms. However, the full potential of these data has not been fully exploited.

We are currently studying the feasibility of utilizing digital radar data for severe weather forecasting and hydrologic applications. The purpose of this paper is to preview some of our experiments and to report an analysis technique that presents a new dimension in the analysis of radar data.

2. DATA

Digital radar data used in this study were furnished by NSSL. The techniques used in the collection, processing, and recording of these data have been described in detail by Wilk et al. (1967). These data are presented in digital form for each 2° of azimuth at 1 n.mi. ranges for succe sive elevation angles in steps of one beam width (2°). B using the calibration data supplied in the digital recor one can convert these data to normalized power or rad reflectivity. It is frequently desirable to convert the data from the radar coordinate system to a system havin the vertical direction as a coordinate. As part of our worl we have studied various coordinate systems, interp lation procedures, and grid intervals that will be reporte on in a future paper. The analyses presented in this pap were performed by use of a quadratic interpolation pr cedure in a cylindrical coordinate system having a 2°×1 n.m ×5,000-ft grid interval. This coordinate system minimiz errors since interpolation is required only for the vertic coordinate.

3. CAZM PRESENTATIONS

If digital radar data are available for successiv elevation angles in steps of one beam width, it is possibl to construct constant altitude reflectivity maps (CAZM for any desired level within the range of the data (figs 1–3). On these maps, radar reflectivity, Z, values ar expressed in dbZs; that is, the value plotted is 10 log Z where Z is expressed in $m^6 \cdot m^{-3}$. These maps are simila to the constant altitude plan position indicator (CAPPI presentations developed by Marshall at McGill Universit (Wein 1963). CAZM presentations in figures 1–3 clearly illustrate the storm intensity at three selected levels thus allowing a three-dimensional interpretation of the echoes.[1] These presentations are very useful in mesoanalysis and/or the study of thunderstorm dynamics. Although a CAZM illustrates the echo or storm intensity at various constant levels, to identify precisely the most intense echoes, one must look at the CAZM for each level and integrate mentally the intensities through the depth of the storm. An analysis technique and display that presents this three-dimensional characteristic in a two-dimensional display is presented in the next section.

[1] Technically, figure 1 is not a CAZM but is a map of 0° reflectivity.

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ATTACHMENT 1

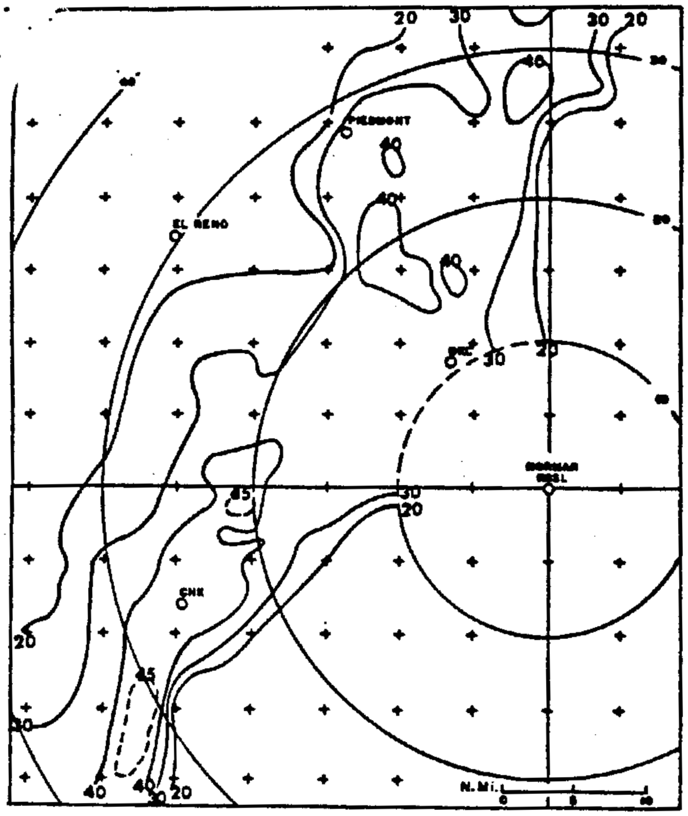

FIGURE 1.—Zero-degree reflectivity in dbZs for 1650 CST, Apr. 26, 1969.

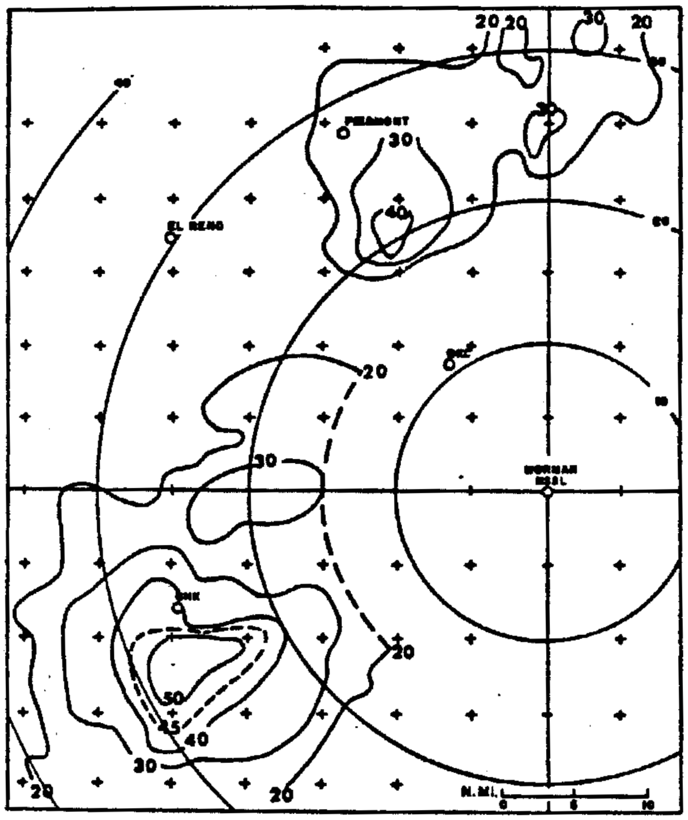

FIGURE 3.—CAZM at 30,000 ft for 1650–57 CST, Apr. 26, 1969

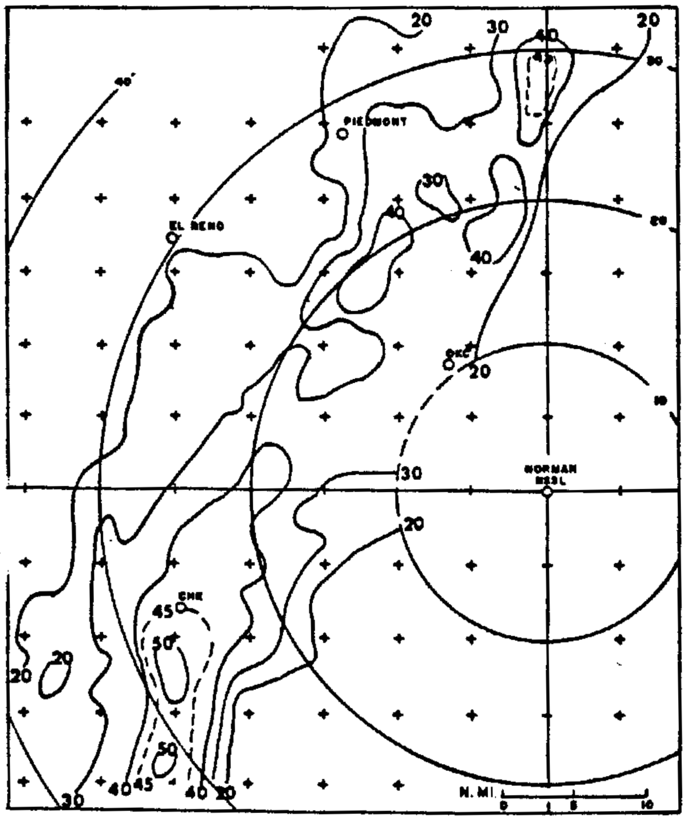

FIGURE 2.—CAZM at 10,000 ft for 1650–57 CST, Apr. 26, 1969.

4. VERTICALLY INTEGRATED LIQUID-WATER CONTENT (VIL)

The concentration of liquid water in a cloud is of considerable meteorological importance. Its magnitude and spatial distribution are important factors in the study of cloud dynamics since they indicate the degree of condensation and development that has taken place. Changes in the water content are important thermodynamically because they are accompanied by large energy changes (Mason 1957). Unfortunately, at this time there is no method of rapidly and accurately measuring the magnitude of liquid-water content; however, its relative magnitude and distribution may be determined by radar measurements if certain assumptions are made regarding the in-cloud drop-size distribution. An exponential drop-size distribution proposed by Marshall and Palmer (1948) seems to fit the distributions observed by several investigators. This distribution is given by

$$n(a)=N_0 \exp(-ba) \tag{1}$$

where a is the drop diameter, $n(a)$ is the number of drops of diameter a, and N_0 and b are parameters in the distribution.

To use radar as an indicator of liquid-water content, M, a relationship was obtained between M and radar reflectivity, Z. Mathematically, M and Z may be defined by

$$M=\frac{\rho_w \pi}{6}\int_0^\infty n(a)a^3 da \tag{2}$$

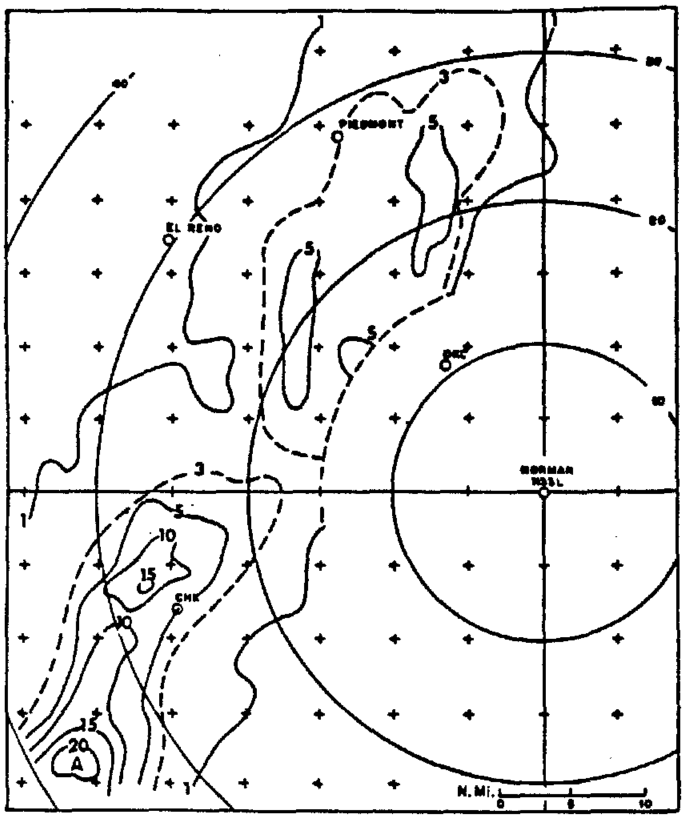

FIGURE 4.—VIL map for 1640–47 CST, Apr. 26, 1969. Isopleths of M^* are in kg·m^{-2}.

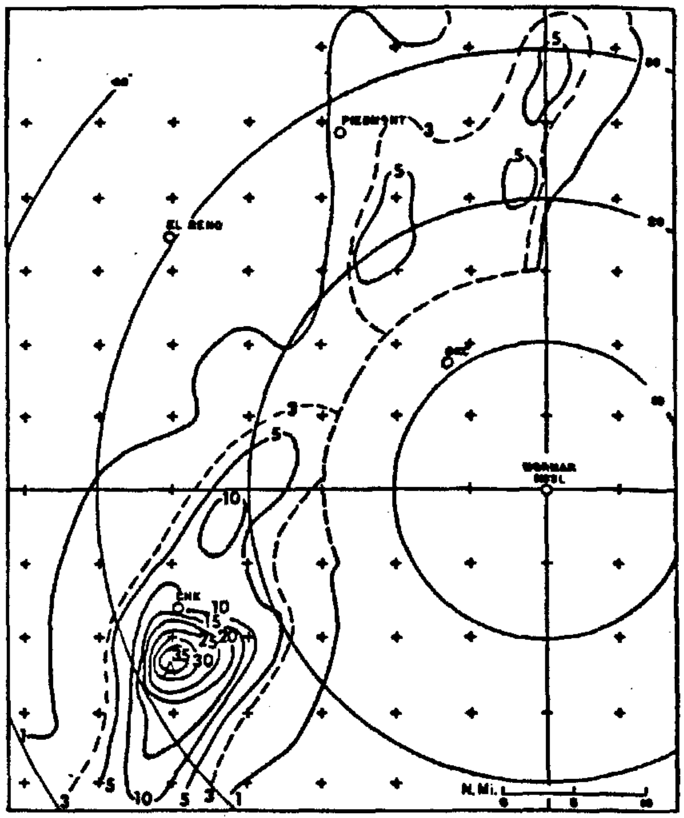

FIGURE 5.—VIL map for 1650–57 CST, Apr. 26, 1969. Isopleths of M^* are in kg · m^{-2}.

and

$$Z=\int_0^x n(a)a^6 da \tag{3}$$

where x is the maximum drop diameter and ρ_w is the density of water. When the Marshall-Palmer drop-size distribution is used in eq (2) and (3), the error is small if the upper limit of integration, x, is replaced by ∞. Integration yields

$$M=\frac{N_0\rho_w\pi}{6}\int_0^\infty \exp(-ba)\, a^3 da=\frac{N_0\rho_w\pi}{6}\frac{\Gamma(4)}{b^4}=\frac{N_0\rho_w\pi}{b^4} \tag{4}$$

and

$$Z=N_0\int_0^\infty \exp(-ba)\, a^6\, da=\frac{N_0\Gamma(4)}{b^7}=\frac{720 N_0}{b^7}. \tag{5}$$

Eliminating the parameter b between eq (4) and (5) yields

$$M=\frac{N_0\pi\rho_w}{[720\times 10^{18} N_0]^{4/7}} Z^{4/7}. \tag{6}$$

For $N_0=8\times 10^6 \text{m}^{-4}$ and $\rho_w=10^6$ g/m^3,

$$M=3.44\times 10^{-3} Z^{4/7} \tag{7}$$

where the units of M are g·m^{-3} and of Z are mm^6·m^{-3}. The factor of 10^{18} in the denominator in eq (6) is required to convert the units of Z from m^6·m^{-3} as given in eq (5) to mm^6·m^{-3}.

M^* is defined as the vertically integrated liquid-water content of the storm and has units of mass per unit area. M^* is computed by integrating eq (7) from the base to the top of the echo; that is,

$$M^*=\int_{h_{base}}^{h_{top}} M dh'=3.44\times 10^{-6}\int_{h_{base}}^{h_{top}} Z^{4/7} dh' \tag{8}$$

where h' is the height expressed in meters and M^* has units of kg·m^{-2}. It should be noted that M and M^* represent the mass of raindrops in a unit volume and unit area, respectively. Since M^* is based on the relationship between M and Z, it would be incorrect to assume that M^* denotes all the in-cloud liquid water. Clouds containing a large number of small drops produce very small values of Z, which may be below the detectable signal of the WSR-57 radar, thus some liquid-water content, M, will not be detected. Hail may also produce fictitious values of liquid water due to enhanced radar return. However, this may be beneficial as an indicator of the severity of a storm.

VIL charts computed from digital radar data collected by NSSL during a storm event on Apr. 26, 1969, are presented in figures 4 and 5. The 5-min isohyetal maps corresponding to these times are presented in figures 6 and 7. The time of the isohyetal map in figure 7 corresponds to the 0° elevation Z map (fig. 1) and the VIL on figure 5. Since the VIL maps integrate over all levels, the configuration of echoes in figures 1–3 and that in figure 5 will be somewhat different. Values of M^* below 1 kg·m^{-2} were not depicted in figures 4 and 5. Visual comparison of these figures indicates that the VIL is possibly a better indicator of rainfall than 0° radar reflectivity. Detailed studies are currently being made to investigate the correlation of rainfall rate, R, with M^*.

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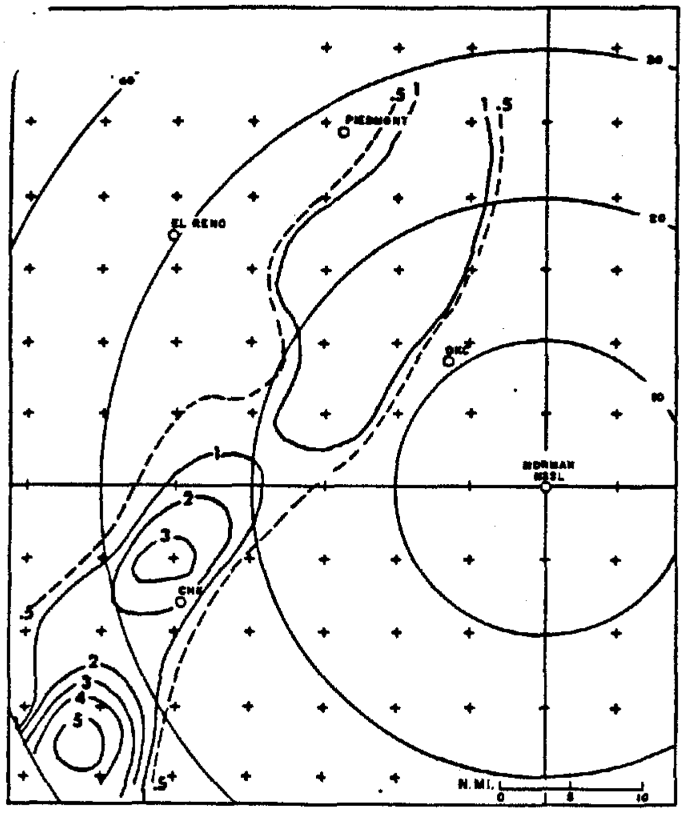

FIGURE 6.—Five-min rainfall rate, R (in./hr), for 1640–45 CST, Apr. 26, 1969.

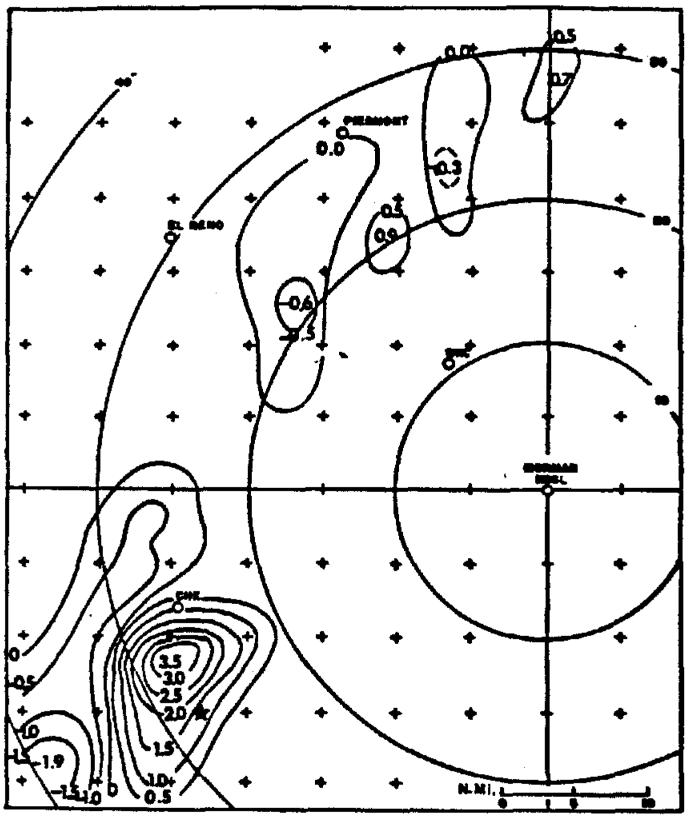

FIGURE 8.—Total change in M^* between 1640 and 1650 CST, Apr. 26, 1969. Isopleths are in kg · m^{-2} · min^{-1}. The star indicates the approximate location of a confirmed tornado.

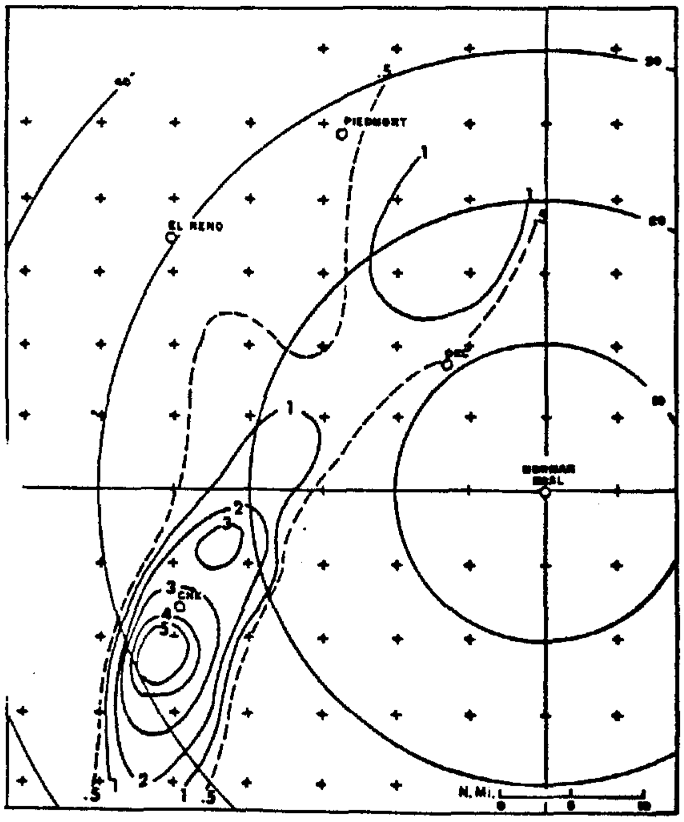

FIGURE 7.—Five-min rainfall rate, R (in./hr), for 1650–55 CST, Apr. 26, 1969.

VIL analyses also provide a new means for identification and possible forecasting of severe storms. The local change in M^* (i.e., $\partial M^*/\partial t$) for the time interval between the maps in figures 4 and 5 is shown in figure 8. It is of interest to note that, in a 10-min interval, the maximum M^* in echo A (figs. 4, 5)i ncreased from 20 kg·m^{-2} to 35 kg·m^{-2}. This rapid increase in liquid-water content appears to be an indicator of "explosive development" of severe storms. At 1700 CST, just after this marked increase in M^*, a confirmed tornado occurred at the location of the star in figure 8. This suggests that the trend in M^* may be an indicator of severe weather development. Other cases with tornado occurrences are being studied to test this hypothesis.

There are many possible applications of the VIL analyses. For example, M^* may be computed from tilt digital data obtained from the national network of radar stations and a composite VIL formed. This composite would have many advantages over the present National Radar Summary Chart (NWS) because it would present an integrated three-dimensional display depicting the character and intensity of all storms in the network. The temporal nature of storm systems can be indicated by successive VIL's or by maps of $\partial M^*/\partial t$ similar to figure 8. This would approach the ultimate goal of the National Radar Network Display.

Although a large-scale digital computer was used to produce the VIL maps presented in this paper, with proper ordering of data, M^* values could be computed would be readily available for real-time use.

Another advantage of vertically integrated values is that vertical integration will filter out strong radar returns that may be due to terrain features or nonstandard propagation. Although these returns may be very strong at low elevation angles, thus adversely affecting present *Z–R* relationships, they become insignificant when integrated over the vertical extent of the storm.

5. CONCLUSIONS

This preliminary investigation indicates that radar tilt data collected over short time intervals may prove to be very beneficial in both hydrologic and severe storm analyses. It is apparent that data collected at constant low antenna elevation angles may not reveal the complete character of a storm. A measure, such as total liquid water, yields an integrated morphology of severe storm systems.

ACKNOWLEDGMENTS

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OFCM

OFFICE OF THE FEDERAL COORDINATOR FOR
METEOROLOGICAL SERVICES AND SUPPORTING RESEARCH

FEDERAL METEOROLOGICAL HANDBOOK NO. 11

DOPPLER RADAR METEOROLOGICAL OBSERVATIONS

PART B

DOPPLER RADAR THEORY AND METEOROLOGY

FCM-H11B-1990
(Interim Version One)

Washington, D.C.
June 1990

ATTACHMENT 2 (9)

Hail Z-R relationships depend upon the stone density, i.e., whether growth has been dry or wet, and the thickness of water films. Douglas (1963) found:

Wet growth $$Z = 84000R^{1.29} \quad (5\text{-}14)$$

Dry growth $$Z = 22500R^{1.17} \quad (5\text{-}15)$$

Considering all of the above, the coefficient and exponent in the Z-R relationship used with the WSR-88D hydrologic software are adaptable parameters with default values set at:

$$Z = 300R^{1.4} \quad (5\text{-}16)$$

Figure 5-1 gives examples of Z-R relationships for various forms of precipitation. As can be seen in Figure 5-1, use of Eq. (5-16) with the WSR-88D hydrologic software should provide a good average for different precipitation types.

The coefficient chosen in Eq. (5-16) is not as critical as one may expect, since a mean bias adjustment by the hydrologic software, using gage data, will be made. Of course this assumes that a reasonable number of rain gages are available under the radar umbrella to satisfactorily remove the mean bias. Also, assuming that the mean bias is removed, the error associated with the exponent is not excessive if a nominal value is chosen (e.g., 1.4) since errors caused by the Z-R relationship tend to cancel as data are averaged over greater space and time scales as shown in Figure 5-2 (Hudlow and Arkell, 1978).

Physical mechanisms that can alter particle size distribution and, consequently, a Z-R relationship include: evaporation, accretion, coalescence, breakup, size sorting, and vertical wind motion. Non-spherical ice particles and the flattening of raindrops as their size increases can enhance or reduce radar reflectivity measurements several decibels, depending on the radar polarization, and contribute errors to estimates of the precipitation rate. Mixed precipitation types, e.g., rain and hail in thunderstorms, can significantly alter a Z-R relationship. One means of minimizing the hail effect is to impose a maximum threshold on the precipitation rate. The threshold should be based on a maximum precipitation rate that can be expected in a given area. The presence of radar echoes beyond the threshold would then indicate the probability of hail and that specified upper limits of precipitation rate should not be exceeded.

Attempts have been made to determine Z-R variations based on other meteorological information such as storm type and weather conditions; however, limited benefit was derived for reducing precipitation rate uncertainty (Stout and Mueller, 1968).

5.3.3 Time and Space Averaging. WSR-88D data are obtained by scanning in azimuth at a series of low elevation angles and making measurements at discrete range and angular intervals. The equivalent reflectivity factor values are converted to rainfall rate with an appropriate Z-R relationship and accumulated in time to yield a spatial distribution of precipitation depth.

Regardless of the Z-R relationship used, this procedure results in time and space sampling errors. Figure 5-3 illustrates the increase in these errors as the sampling interval is increased over the various averaging areas. For example, the top graph in Figure 5-3 shows for this data set that

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