Moment of Inertia of passengers:

The moment of Inertia for passengers was estimated using the program MATHCAD. The calculations are outlined in the following sections. Because of uncertainties in seating position two estimates were made in order to gauge the range of values. In the first estimate passengers are placed closer to the cg than in the second estimate. The lower of the two estimates was used for the simulation, however, the difference between the two estimates is relatively small compared to the overall yaw inertia of the bus and therefore the choice would make little difference in the outcome of the simulation

Seating and Passenger Information Used in Calculations

The passenger and seating information used in this calculation was compiled in the Survival Factors Report. This information is summarized in the Survival Factors Seating Chart shown in Figure 1.

Weights used in calculations

Estimated weight for passengers was based on the ^{\$0}Opercentile data published by American Journal of Clinical Nutrition, Vol 32, Pages (607 -629), 1979, which takes into account both age and gender in its computations of the 50th percentiles Childs weight. Adult data was taken from the same source. The weights by age are summarized below:

AGE	Weight
6	46
7	50
8	56
Adult	152

Table 1 – Passenger Weight Data

AGE	WEIGHT
7	47
Adult	125

Assumptions used in calculation

- 1. The occupant is 2/3 or approximately 10 inches rearward of the front of the seat (7 inches rearward of the seat location shown on miniatures seat chart)
- 2. The calculation assumes that the seats on the left are even with the seats on the right
- 3. The yaw moment of inertia of the passenger about his center of gravity is Attachment B Report Page29

negligible compared with his moment about the cg of the bus.

Lateral Seating position

The center aisle of the bus is 15 inches wide. Passenger seats are 39 inches wide with three seating positions each approximately 13 inches wide. Based on this information the lateral seating position of the passenger seated next to the center aisle seat is approximately 14" lateral of the midpoint of the bus. The center passenger seating position is 27" inches lateral of the midpoint. The outboard passenger seating position is approximately 40" lateral of the centerline of the bus.

Longitudinal Seating Position

The longitudinal seating position was based on the manufacturers seating diagram and is summarized by a matrix in the MATHCAD calculations. The occupant was considered to be about 10 inches rearward of the front of the seat (7 inches rearward of the seat location shown on the manufacturers diagram)

Passenger Seating Positions

Due to uncertainty in seating position two estimates of yaw moment of inertia were made. The primary difference between the two estimates is that in the first estimate 8 passengers listed as unknown in the Survival Factors diagrams (Figure 1) are placed closer to the cg of the bus than in the second estimate. Estimates were performed using the program MATHCAD. MATHCAD inputs and outputs for both estimates are included at the end of this Attachment.

In the first estimate, passengers in the Figure 1 with known age, gender, and seating position were placed in their designated seating position. Six of the female passengers listed as "unknown" were placed in the six seats designated with "girl" in the Survival Factors Seating Chart (Figure 1) The two boys listed as "unknown" in Figure 1 were placed in the two seats closest the center aisle in row 9, this row had two seats designated as boy but it was unknown which two seats in the row the boys occupied. The remaining 6 girls (also listed under unknown in Figure 1) were placed in the unoccupied seats closest to the cg of the bus.

In the second estimate passengers in Figure 1 with known age, gender, and seating position were placed in their designated seat. Six of the unknown female passengers were placed in the six seats designated with girl in the Survival Factors Seating Chart (Figure 1). The two boys listed as unknown in Figure 1 were place in the two seats furthest from the center aisle in row 9. The remaining 6 girls (also listed under "unknown") were placed in the unoccupied seats furthest from the cg of the bus (see Figure 4).

Passengers weights Used in Estimates

If the passenger seating position, age, and gender were given in Figure 1 the weight was taken from Table 1 based on age and gender. If the passenger was female and her seating position was unknown the average weight of the females listed as unknown in figure 1 was used. If the passenger was male passenger and his seating position was unknown then the average weight of the male passengers listed as unknown was used. (In calculating the average weight of unknowns the data in table 1 was utilized. The age and gender of passengers with unknown seating positions is given in Figure 1)

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Driver and Adults in Middle Aisle

For convenience the driver and adults seated in the middle aisle were neglected in the inertia calculations. The effect of this on the overall moment of inertia would be small.

Summary of Results

First Estimate Unknown Passengers Closest to the CG

30	6 passengers with seat, gender and age designation:	
	······································	97,170lbsec in
6	unknown females placed in seats designated with girl:	
		11,600 lbsec in
8	Unknowns: six females seated closest to the cg, two ma Seated closest to the center aisle in row 9	les
	······	11,610 lbsec in
2	Adult passengers seated in center aisle	0 lbsecin
1	Adult driver	0 lbs ² ecin
	Total	

Second Estimate Unknown Passengers Furthest from the CG

36	passengers with seat, gender and age designation:	
		97,170 lbs ² ec in
6	Unknown females placed in seats designated with girl:	
	- 	11,600 lbsec in
8	Unknowns: six females seated furthest to the cg, two mal Seated furthest the center aisle in row 9	es
		24,110 lbsec in
2	Adult passengers seated in center aisle	0 lbsecin
1	Adult driver	0 lbsecin
	Total	132,880 lbsecin



Figure 1 - Survival Factors Seating Diagram

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Figure 2 – Seating positions of occupant with known age, gender and seating position.

MATHCAD Calculations for Distances from seat to CG of Bus

(-40 -27 -14 14 27 40)	
-40 -27 -14 14 27 40	
-40 -27 -14 14 27 40	
-40 -27 -14 14 27 40	
-40 -27 -14 14 27 40	
-40 -27 -14 14 27 40	
-40 -27 -14 14 27 40	
-40 -27 -14 14 27 40	
-40 -27 -14 14 27 40	
-40 -27 -14 14 27 40	
(-40 -27 -14 14 27 40)	
	$\begin{pmatrix} -40 & -27 & -14 & 14 & 27 & 40 \\ -40 & -27 & -14 & 14 & 27 & 40 \\ -40 & -27 & -14 & 14 & 27 & 40 \\ -40 & -27 & -14 & 14 & 27 & 40 \\ -40 & -27 & -14 & 14 & 27 & 40 \\ -40 & -27 & -14 & 14 & 27 & 40 \\ -40 & -27 & -14 & 14 & 27 & 40 \\ -40 & -27 & -14 & 14 & 27 & 40 \\ -40 & -27 & -14 & 14 & 27 & 40 \\ -40 & -27 & -14 & 14 & 27 & 40 \\ -40 & -27 & -14 & 14 & 27 & 40 \\ -40 & -27 & -14 & 14 & 27 & 40 \\ -40 & -27 & -14 & 14 & 27 & 40 \\ \end{pmatrix}$

The Y matrix represents the occupant's cg location relative to the centerline of the bus.

 $Y := Y \cdot in$ Makes units of Y matix inches.

```
14.8 14.8 14.8 14.8 14.8 14.8
      42.4 \quad 42.4 \quad 42.4 \quad 42.4 \quad 42.4 \quad 42.4
       70
             70
                   70
                         70
                               70
                                     70
      97.6 97.6 97.6 97.6 97.6 97.6
     125.2 125.2 125.2 125.2 125.2 125.2
X:= 152.8 152.8 152.8 152.8 152.8 152.8
      180.4 \ 180.4 \ 180.4 \ 180.4 \ 180.4 \ 180.4
      208 208 208 208 208 208
     235.6 235.6 235.6 235.6 235.6 235.6
     263.2 263.2 263.2 263.2 263.2 263.2
     290.8 290.8 290.8 290.8 290.8 290.8
```

X is the longitudinal distance from the zero location given in the manufacturers diagram to the seat.

MATHCAD Calculations for Distance From CG of Bus

 $X := X \cdot in$ X matix units are inches

XF := 187.6n Where, XF = the distance from the front bumper of the bus to the cg.

- i := 0..10
- j := 0..5

$$XtoCG_{i,j} := \left[\left(X_{i,j} + 103.3n + 7in - 187.6n \right)^2 + \left(Y_{i,j} \right)^2 \right]^{.5}$$

XtoCG is the linear distance from the seat postion to the cg of the bus. X is defined above as distance between the zero point and the seat position in the manufacturers diagram. 103.3 is the distance from the zero point on the manufacturers diagram to the front bumper. Seven inches is the estimated distance from the front of the seat to the passengers cg. Y is the distance from the seating postion to the centerline of the bus

		0	1	2	3	4	5
	0	74.204	68.083	64.049	64.049	68.083	74.204
	1	53.085	44.125	37.603	37.603	44.125	53.085
	2	40.661	27.969	15.789	15.789	27.969	40.661
	3	44.856	33.78	24.659	24.659	33.78	44.856
VtoCG-	4	62.405	54.986	49.904	49.904	54.986	62.405
AIOCO-	5	85.442	80.183	76.787	76.787	80.183	85.442
	6	110.588	106.577	104.046	104.046	106.577	110.588
	7	136.684	133.46	131.448	131.448	133.46	136.684
	8	163.276	160.586	158.918	158.918	160.586	163.276
	9	190.155	187.85	186.426	186.426	187.85	190.155
	10	217.215	215.2	213.959	213.959	215.2	217.215

in

Mathcad calculations for moment of inertia about z-axis for Occupants with known gender, age, and seating position



 $Y := Y \cdot in$

Set units equal to inches

Mathcad calculations for moment of inertia about z-axis for Occupants with known gender, age, and seating position

14.8 14.8 14.8 14.8 14.8 14.8 42.4 42.4 42.4 42.4 42.4 42.4 70 70 70 70 70 70 97.6 97.6 97.6 97.6 97.6 97.6 125.2 125.2 125.2 125.2 125.2 125.2 X:= 152.8 152.8 152.8 152.8 152.8 152.8 $180.4 \ 180.4 \ 180.4 \ 180.4 \ 180.4 \ 180.4$ 208 208 208 208 208 208 208 235.6 235.6 235.6 235.6 235.6 235.6 263.2 263.2 263.2 263.2 263.2 263.2 290.8 290.8 290.8 290.8 290.8 290.8

Longitudinal distance from seat to 0 in manufacturers diagram.

 $X := X \cdot in$ Set units equal to inches

XF:=187.6n

i := 0..10

j:=0..5

$$I_{i,j} := \left[\left[\left(X_{i,j} + 103.3n + 7in - 187.6n \right)^2 + \left(Y_{i,j} \right)^2 \right] \right] \cdot \frac{W_{i,j}}{g}$$

XtoCG is the linear distance from the seat postion to the cg of the bus. X is defined above as distnance between the zero point and the seat position in the manufacturers diagram . 103.3is the distance from the zero point on the manufacturers diagram to the front bumper. Seven inches is the estimated distance from the front of the seat to the passengers cg.

		0	1	2	3	4	5	
	0	670.297	0	499.383	0	0	0	
	1	343.047	0	457.8	457.8	252.146	408.737	
	2	214.108	0	32.284	36.158	0	201.261	
	3	260.574	147.776	78.75	74.025	0	239.728	
I –	4	0	0	806.296	361.221	0	504.342	$1b \sec^2 in$
1 -	5	0	0	0	0	0	0	ibsec in
	6	1.774·10 ³	1.648·10 ³	1.402·10 ³	0	0	1.774·10 ³	
	7	0	0	0	0	0	0	
	8	3.452·10 ³	3.139·10 ³	8.177·10 ³	0	0	0	
	9	4.683·10 ³	4.57·10 ³	0	1.368·10 ⁴	4.296·10 ³	4.402 [.] 10 ³	
	10	6.11·10 ³	0	5.454·10 ³	1.482·10 ⁴	5.997·10 ³	5.744·10 ³	

Mathcad calculations for moment of inertia about z-axis for Occupants with known gender, age, and seating position

$$I := \sum_{i=0}^{10} \sum_{j=0}^{5} I_{i,j}$$

Total moment of inertia about the z-axis of bus for occupants with known sex, age and seating postions

 $I = 9.717 \times 10^4 lb sec^2 in$

Mathcad calculations for moment of inertia about z-axis for Six seating positions designated "girl" with no age

	(0	0	0	0	0	0)
	0	0	0	0	0	0
	0	0	0	0	0	0
	0	0	0	0	0	0
	0	0	0	0	0	0
W :=	0	0	0	0	0	0
	0	0	0	47.6	47.6	0
	47.6	0	47.6	47.6	0	47.6
	0	0	0	0	0	0
	0	0	0	0	0	0
	0	0	0	0	0	0)

Weight of occupant. Zero means either no occupant
or that occupant in that seat is not included in this
portion of the estimate.
The girl's seating postion is shown in Figure 1 of
this Attachment.

$$W := W \cdot lb$$
 Set units equal to lbs for matrix

$$Y := \begin{pmatrix} -40 & -27 & -14 & 14 & 27 & 40 \\ -40 & -27 & -14 & 14 & 27 & 40 \\ -40 & -27 & -14 & 14 & 27 & 40 \\ -40 & -27 & -14 & 14 & 27 & 40 \\ -40 & -27 & -14 & 14 & 27 & 40 \\ -40 & -27 & -14 & 14 & 27 & 40 \\ -40 & -27 & -14 & 14 & 27 & 40 \\ -40 & -27 & -14 & 14 & 27 & 40 \\ -40 & -27 & -14 & 14 & 27 & 40 \\ -40 & -27 & -14 & 14 & 27 & 40 \\ -40 & -27 & -14 & 14 & 27 & 40 \\ -40 & -27 & -14 & 14 & 27 & 40 \\ \end{pmatrix}$$

Lateral seating distance for occupants

$Y := Y \cdot in$ Set units equal to inches

Mathcad calculations for moment of inertia about z-axis for Six seating positions designated "girl" with no age

14.8 14.8 14.8 14.8 14.8 14.8 42.4 42.4 42.4 42.4 42.4 42.4 70 70 70 70 70 70 97.6 97.6 97.6 97.6 97.6 97.6 125.2 125.2 125.2 125.2 125.2 125.2 X:= 152.8 132.8 132.8 132.8 132.8 132.8 180.4 180.4 180.4 180.4 180.4 180.4 208 208 208 208 208 208 235.6 235.6 235.6 235.6 235.6 235.6 263.2 235.6 235.6 235.6 235.6 235.6 290.8 290.8 290.8 290.8 290.8 290.8

Distance from seat to 0 in manufacturers diagram.

 $X := X \cdot in$ Set units equal to inches

XF := 187.6n

i := 0..10

j := 0..5

$$I_{i,j} := \left[\left[\left(X_{i,j} + 103.3n + 7in - 187.6n \right)^2 + \left(Y_{i,j} \right)^2 \right] \right] \cdot \frac{W_{i,j}}{g}$$

XtoCG is the linear distance from the seat postion to the cg of the bus. X is defined above as distnance between the zero point and the seat position in the manufacturers diagram . 103.3is the distance from the zero point on the manufacturers diagram to the front bumper. Seven inches is the estimated distance from the front of the seat to the passengers cg.

		0	1	2	3	4	5	
	0	0	0	0	0	0	0	
	1	0	0	0	0	0	0	
	2	0	0	0	0	0	0	
	3	0	0	0	0	0	0	
т —	4	0	0	0	0	0	0	$1b\cos^2 in$
1 –	5	0	0	0	0	0	0	ID sec III
	6	0	0	0	1.335 [.] 10 ³	1.4 10 ³	0	
	7	2.303·10 ³	0	2.13 [.] 10 ³	2.13 [.] 10 ³	0	2.303·10 ³	
	8	0	0	0	0	0	0	
	9	0	0	0	0	0	0	
	10	0	0	0	0	0	0	

Mathcad calculations for moment of inertia about z-axis for Six seating positions designated "girl" with no age

$$I := \sum_{i \ = \ 0}^{10} \ \sum_{j \ = \ 0}^{5} \ I_{i, \, j}$$

 $I = 1.16 \times 10^4 \, \text{lb} \, \text{sec}^2 \, \text{in}$

Total moment of inertia about the z-axis for 6 seating positions in Figure 1 with sex listed as girl, a known seating postion and no given age.

Error! Not a valid link

Figure 3 – Eight unknown occupants seated in positions closest to the CG. Circles designate the females. Males by squares. Distances from cg to all seats with no designated age sex or gender (the two boys in row 9 were not assigned a specific seat in row 9) are given.

Mathcad calculations for moment of inertia about z-axis of bus for 8 uknown occupants seated closest to the cg

	(0)	0	0	0	0	0)
	0	47.6	0	0	0	0
	0	047.6	0	0	047.6	0
	0	0	0	0	47.6	0
	0	47.6	0	0	047.6	0
W :=	0	0	0	0	0	0
	0	0	0	0	0	0
	0	0	0	0	0	0
	0	0	0	53	53	47.6
	0	0	0	0	0	0
	0	0	0	0	0	0)

Weight of occupant for the eight occupants. This matix resembles information from Figure 3 of this Attachment. The boys in row 9 are seated nearest the center since this calculation is intended to check the lower bound of the moment of ineria,



Set units equal to lbs for matrix

	Lateral seating distance for occupants
-40 -27 -14 14 27 $40-40$ -27 -14 14 27 40	

 $Y := Y \cdot in$

Set units equal to inches

Mathcad calculations for moment of inertia about z-axis of bus for 8 uknown occupants seated closest to the cg

$$X := \begin{pmatrix} 14.8 & 14.8 & 14.8 & 14.8 & 14.8 & 14.8 \\ 42.4 & 42.4 & 42.4 & 42.4 & 42.4 & 42.4 \\ 70 & 70 & 70 & 70 & 70 & 70 \\ 97.6 & 97.6 & 97.6 & 97.6 & 97.6 & 97.6 \\ 125.2 & 125.2 & 125.2 & 125.2 & 125.2 \\ 152.8 & 132.8 & 132.8 & 132.8 & 132.8 \\ 180.4 & 180.4 & 180.4 & 180.4 & 180.4 \\ 208 & 208 & 208 & 208 & 208 \\ 235.6 & 235.6 & 235.6 & 235.6 & 235.6 \\ 263.2 & 235.6 & 235.6 & 235.6 & 235.6 \\ 290.8 & 290.8 & 290.8 & 290.8 & 290.8 \\ 290.8 & 290.8 & 290.8 & 290.8 & 290.8 \\ 290.8 & 290.8 & 290.8 & 290.8 & 290.8 \\ \end{pmatrix}$$

Distance from seat to 0 in manufacturers diagram.

 $X := X \cdot in$ Set units equal to inches

XF := 187.6n

$$i := 0..10$$

j:=0..5

$$I_{i,j} := \left[\left[\left(X_{i,j} + 103.3n + 7in - 187.6n \right)^2 + \left(Y_{i,j} \right)^2 \right] \right] \cdot \frac{W_{i,j}}{g}$$

XtoCG is the linear distance from the seat postion to the cg of the bus. X is defined above as distance between the zero point and the seat position in the manufacturers diagram. 103.3is the distance from the zero point on the manufacturers diagram to the front bumper. Seven inches is the estimated distance from the front of the seat to the passengers cg.

Mathcad calculations for moment of inertia about z-axis of bus for 8 uknown occupants seated closest to the cg

		0	1	2	3	4	5	[
	0	0	0	0	0	0	0	
	1	0	240.043	0	0	0	0	•
	2	0	96.447	0	0	96.447	0	•
	3	0	0	0	0	140.683	0	•
I _	4	0	372.75	0	0	372.75	0	$1b\cos^2 in$
1 =	5	0	0	0	0	0	0	· IU SEC III
	6	0	0	0	0	0	0	
	7	0	0	0	0	0	0	•
	8	0	0	0	3.467·10 ³	3.54·10 ³	3.287·10 ³	
	9	0	0	0	0	0	0	•
	10	0	0	0	0	0	0	

$$I := \sum_{i = 0}^{10} \sum_{j = 0}^{5} I_{i,j}$$

 $I = 1.161 \times 10^4 lb sec^2 in$

Total moment of inertia about the z-axis for 8 uknov occupants seated in postions nearest the bus cg



Figure 4 – Eight unknown occupants seated in positions furthest from the CG. Circles designate the females. Males by squares. Distances from cg to all seats with no designated age sex or gender are listed (the two boys in row 9 were not assigned a specific seat in row 9 so therefore the distances to these seats are shown).

Mathcad calculations for moment of inertia about z-axis of bus for 8 uknown occupants seated furthest from the cg

	0	0	0	0	0	0)	
	0	0	0	0	0	0	
	0	0	0	0	0	0	
	0	0	0	0	0	0	
	0	0	0	0	0	0	
W :=	0	0	0	0	0	47.6	
	0	0	0	0	0	0	
	0	47.6	0	0	47.6	0	
	0	0	0	47.6	53	53	
	0	0	47.6	0	0	0	
	0	47.6	0	0	0	0)	

Weight of occupant for the eight occupants. This matix resembles information from Figure 4 of this Attachment. The boys in row 9 are seated furthest from the center since this calculation is intended to check the upperr bound of the moment of ineria,

 $W := W \cdot lb$ Set units equal to lbs for matrix

Y :=		Lateral seating distance for occupants
	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	

$$Y := Y \cdot in$$

Set units equal to inches

<u>Mathcad calculations for moment of inertia about z-axis of bus for</u> <u>8 uknown occupants seated furthest from the cg</u>

```
14.8 14.8 14.8 14.8 14.8 14.8
     42.4 42.4 42.4 42.4 42.4 42.4
      70
           70
                 70
                       70
                            70
                                  70
     97.6 97.6 97.6 97.6 97.6 97.6
     125.2 125.2 125.2 125.2 125.2 125.2
X := 152.8 132.8 132.8 132.8 132.8 132.8
     180.4 180.4 180.4 180.4 180.4 180.4
     208 208 208 208 208 208
     235.6 235.6 235.6 235.6 235.6 235.6
     263.2 235.6 235.6 235.6 235.6 235.6
     290.8 290.8 290.8 290.8 290.8 290.8
```

Distance from seat to 0 in manufacturers diagram.

 $X := X \cdot in$ Set units equal to inches

XF := 187.6n

i := 0..10

j := 0..5

$$I_{i,j} := \left[\left[\left(X_{i,j} + 103.3n + 7in - 187.6n \right)^2 + \left(Y_{i,j} \right)^2 \right] \right] \cdot \frac{W_{i,j}}{g}$$

XtoCG is the linear distance from the seat postion to the cg of the bus. X is defined above as distnance between the zero point and the seat position in the manufacturers diagram . 103.3is the distance from the zero point on the manufacturers diagram to the front bumper. Seven inches is the estimated distance from the front of the seat to the passengers cg.

Mathcad calculations for moment of inertia about z-axis	of bus for
8 uknown occupants seated furthest from the cg	

		0	1	2	3	4	5	
	0	0	0	0	0	0	0	
	1	0	0	0	0	0	0	
	2	0	0	0	0	0	0	
	3	0	0	0	0	0	0	
I —	4	0	0	0	0	0	0	$1bcoc^2$ in
1 =	5	0	0	0	0	0	577.018	iusee m
	6	0	0	0	0	0	0	
	7	0	2.196·10 ³	0	0	2.196·10 ³	0	
	8	0	0	0	3.114·10 ³	3.54·10 ³	3.66·10 ³	
	9	0	0	3.114·10 ³	0	0	0	
	10	0	5.71·10 ³	0	0	0	0	

$$I := \sum_{i \ = \ 0}^{10} \ \sum_{j \ = \ 0}^{5} \ I_{i, j}$$

 $I = 2.411 \times 10^4 lb sec^2 in$

Total moment of inertia about the z-axis for 8 uknown occupants seated in postions in undesignated seats furthest from the bus cg